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APOLLONII PERGÆI

INCLINATIONUM

LIBRI DUO.

RESTITUEBAT

SAMUEL HORSLEY, R.S.S.

6 H d

OXONII,

E TYPOGRAPHEO CLARENDONIANO.

M. DCC. LXX.

Proſtant Venales apud DAN. PRINCE, *Oxonii.* J. RIVINGTON, J. NOURSE, P. ELMSLEY, & B. WHITE, *Londini.* J. WOODYER, *Cantabrigiæ.*

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BRITISH MUSEUM
NATURAL HISTORY

LECTORI:

QUÆ ſequuntur pleraque indaginis ſunt noſtræ. Aliunde deſumpta auctoribus ſuis quæque retulimus, ne laudis alienæ videremur appetentes.

Tyrones ad hæcce ne accedant, priuſquam Euclidis Data ad unguem uſque percalluerint, & Apollonii Libros, de Rationem & Spatium abſcindendo, ab Halleio olim reſtitutos, diligenter evolverint.

Argumentum Operis, e Pappo.

Νεύειν λέγεται γραμμὴ ἐπὶ σημεῖον, ἐὰν ἐπεκβαλλομένη ἐπ' αὐτὸ παραγένηται. Καθόλου δὲ τὸ αὐτό ἐστιν, ἐάν τε ἐπὶ δοθὲν νεύειν σημεῖον λέγηται· ἐάν τέ ἐστί τι ἐπ' αὐτῆς δοθέν· ἐάν τε πρὸς δοθέντι ἐστὶ σημεῖα. Ἐπέγραψαν δὲ ταῦτα Νεύσεις ἀφ' ἑνὸς τῶν εἰρημένων. Προβλήματος δὲ ὄντος καθολικοῦ τούτου, δύο δοθεισῶν γραμμῶν θέσει, θεῖναι μεταξὺ τούτων εὐθεῖαν τῷ μεγέθει δεδομένην, νεύουσαν ἐπὶ δοθὲν σημεῖον, ἐπὶ ταύτης[a], τῶν ἐπὶ μέρους διαφορὰ τὰ ὑποκείμενα ἐχόντων, ἃ μὲν λεῖ ἐπίπεδα, ἃ δὲ στερεὰ[b], ἃ δὲ γραμμικά. Τῶν δὲ ἐπιπέδων ἀποκληρώσαντες τὰ πρὸς πολλὰ χρησιμώτερα, ἔδειξαν προβλήματα ταῦτα. Θέσει δεδομένων ἡμικυκλίου τε καὶ εὐθείας πρὸς ὀρθὰς τῇ βάσει, ἢ δύο ἡμικυκλίων ἐπ' εὐθείας ἐχόντων τὰς βάσεις, θεῖναι δοθεῖσαν τῷ μεγέθει εὐθεῖαν μεταξὺ τῶν δύο γραμμῶν, νεύουσαν ἐπὶ γωνίαν ἡμικυκλίου. Καὶ ῥόμβου δοθέντος, καὶ ἐπεκβληθείσης μόνης μιᾶς πλευρᾶς, ἁρμόσαι ὑπὸ τοῦ ἐκτὸς γωνίαν δεδομένῃ τῷ μεγέθει εὐθεῖαν, νεύουσαν ἐπὶ τοῦ ἀπεναντίον γωνίαν[c]. Καὶ θέσει δοθέντος κύκλου ἐναρμόσαι εὐθεῖαν μεγέθει δεδομένῃ νεύουσαν ἐπὶ δοθέν. Τούτων ἃ ἐν μὲν τῷ πρώτῳ τεύχει δέδεικται, τὸ ἐπὶ δ ἑνὸς ἡμικυκλίου καὶ εὐθείας,

[a] Fortasse ἐπὶ τούτου.

[b] Solida Problemata Antiquis dicta sunt, quæ per Coni sectiones, Linearia quæ non nisi per lineas ortûs Mechanici, puta conchoideas, cissoideas atque id genus alias, resolutionem capiebant.

[c] Sed & Problema de rectâ, magnitudine datâ, angulo *interiori* per oppositum angulum subjiciendâ, ab Apollonio resolutum esse ex Papp. Lib. 7. prop. 73. liquido constat.

ἔχον

ἔχον πλώσεις τεσσάρας [a], καὶ τὸ ἐπὶ ☿ κύκλῳ ἔχον πτώσεις δύο, καὶ τὸ ἐπὶ ☿ ῥόμβῳ πλώσεις ἔχον δύο. Ἐν ᾧ τῷ δευτέρῳ τεύχει, τὸ ἐπὶ ☉ δύο ἡμικυκλίων, ☿ ὑποθέσεως πλώσεις ἐχούσης δέκα [b]. ἐν δὲ ταύταις ὑποδιαιρέσεις πλείονες διορισταὶ, ἕνεκα ☿ δεδομένου μεγέθους ☿ εὐθείας.

[a] Legendum πέντε.

[b] Semicirculorum nempe ſtatus quintuplex, Circulis contingentibus intus, contingentibus extrinſecus, Nullibi occurrentibus, altero incluſo, Nullibi occurrentibus incluſo neutro, Secantibus. In ſtatu autem unoquoque, gemina erit ſemicirculorum poſitio, ad partes baſeos aut eaſdem aut contrarias. En tibi decem, ni fallor Hypotheſeos Caſus, quos Pappus innuit.

INTERPRETATIO.

VERGERE, ſeu inclinari, dicitur linea ad punctum, ſi producta ad ipſum pervenit: univerſim autem idem eſt, ſive dicatur linea inclinari ad datum punctum, ſive punctum aliquod in lineâ dari, ſive per datum punctum lineam tranſire. Ex primâ loquendi formâ, hi Libri *Inclinationes* inſcripti ſunt. Problema vero cum ſit generale, "Duabus lineis poſitione datis, inter "eas inſerere rectam, magnitudine datam, quæ ad da-"tum punctum pertingat," Specialium, quæ in generali illo incluſa ſunt, & ſubjecta diverſa habent, alia plana ſunt, alia ſolida, alia etiam linearia. E planis vero hæcce, quæ ad plura magis utilia viſa ſunt, ſelecta demonſtrarunt. "Datis poſitione ſemicirculo & rec-"tâ, quæ ad Baſin normalis ſit; vel duobus ſemicir-"culis in eâdem rectâ baſes habentibus, inſerere rectam, "magnitudine datam, inter duas illas lineas, quæ ad "angulum ſemicirculi pertingat." Et, "Rhombo "dato, & producto uno ejus latere, adaptare, ſub an-"gulo ejus exteriore, rectam, magnitudine datam, ad an-"gulum oppoſitum vergentem." Et, "in circulo po-"ſitione dato, inſerere rectam, magnitudine datam, quæ "ad datum punctum pertingat." In primo autem libro, demonſtratum eſt Problema de uno ſemicirculo & rectâ; quod quidem quinque Caſus habet: ut & illud de circulo, in duos Caſus diviſum: atque etiam illud

de

de Rhombo, duos quoque Caſus habens. In ſecundo vero habetur unicum problema de duobus ſemicirculis, cujus decem ſunt Hypotheſeos Caſus; atque horum etiam plures ſunt ſubdiviſiones dioriſticæ, quæ datam magnitudinem rectæ inſerendæ reſpiciunt.

ERRATA.

Pag. 5. *lin.* 1. *pro* quæ circumferentiæ BA, rectæque DE, in-, *lege* quæ, ad A vergendo, circumferentiæ BA, rectæque DE, unquam in-.
Pag. 5. *lin.* 24 & 25. *pro* Conſtructio; ad utraſque partes minimæ, *lege* Conſtructio, ad utraſque partes minimæ;
Pag. 16. *lin.* 7. *poſt*-cunt *adde* (Vid. Tab. II. fig. 1. N. 2.)
lin. 14. *pro* N *lege* R.
Pag. 32. *lin. antepenult. pro* ant *lege* aut.
Pag. 38. *lin.* 20. *pro* BF *lege* BE.
Pag. 51. *lin.* 10. *pro* BF *lege* BE.
Pag. 64. *lin.* 15. *pro* K *lege* k.
Pag. 87. *lin. antepenult. pro* fuerit *lege* fuerint.
Pag. 103. *lin.* 26. *pro* in conſulto *lege* inconſulto.
Pag. 106. *lin.* 24. *poſt* datum *adde* magnitudine.

Lemma vero ſecundum, Libri ſecundi, hoc modo corrigatur;

Poſt Dico rectam AB poſitione datam eſſe. *(Pag.* 27. *lin. ult.) adde,* Modo linea DBC circuli Circumferentia non ſit cujus centrum A.

To the Book-binder

Place the three Plates belonging to the firſt Book, facing the blank Page at the End of the firſt Book. And diſpoſe the Plates belonging to the ſecond Book in the following Manner:

Plates I and II facing pag. 48.
Plate III facing pag. 56.
Plates IV, V, VI, and VII, facing pag. 88.
Plate VIII facing pag. 104.
Plates IX and X at the End.

APOLLONII PERGÆI
INCLINATIONUM
LIBER PRIMUS RESTITUTUS.

Problema I.

IN circulo magnitudine & positione dato, rectam datæ magnitudinis aptare, quæ ad datum extra punctum vergat. Vid. Tab. I. fig. 1.

Circulo magnitudine & positione dato ABC inscriptam puta EB datæ magnitudinis, quæ ad punctum datum D extra circulum vergat. Esto F circuli dati centrum. Juncta DF & producta circumferentiæ datæ in punctis A, C, occurrat. Jungatur FE, & ab F ducatur FG, ad inscriptam EB perpendicularis. Æquales erunt BG, GE. Datâ igitur magnitudine BE, data erit semissis ejus, GE. Et data est FE, propter circulum datum. Data est igitur (propter angulum ad G rectum) recta FG magnitudine. Datum vero punctum F. Est igitur punctum G ad circumferentiam positione datam, (quæ sit GMH) dato nimirum centro F, datoque intervallo FG descriptam. Propter angulum ad G rectum, recta DG datam circumferentiam GMH in G contingit. Datum vero punctum D. Recta igitur DG positione data. (per 91. dat.) Q. E. I.

Limes.

Recta inſcribenda diametro circuli non debet eſſe major.

Compositio.

Detur recta N. Junctaque DF, & producta, circumferentiæ in A, C, occurrat; ſi datæ N recta AC æqualis ſit, factum eſt quod fieri oportebat. Si inæquales ſint rectæ N, AC, minor eſto N. Alioqui problema non poterit componi. Et in dato circulo ABC, inſcribatur AP, datæ N æqualis. A centro F ad AP perpendicularem age FQ. Centro F intervallo FQ ſcribatur circulus QGH. Diametro etiam FD deſcribatur circulus GDH, qui circulo GHQ in punctis G, H, occurrat. Junctæ DG, DH, & productæ, circumferentiæ ABCK in punctis B, E, K, L occurrant. Rectæ DB, DK problemati utræque ſatisfaciunt.

Problema II.

In circulo poſitione & magnitudine dato rectam datæ magnitudinis aptare, quæ per datum intra punctum tranſeat.

Analyſis & compoſitio eadem ac præcedentis, limite etiam eodem, ne recta ſcilicet inſcribenda dati circuli diametro major ſit. Vide Tab. I. fig. 2.

Problema III.

Semicirculo magnitudine & poſitione dato, datâque poſitione rectâ ad baſin perpendiculari, datæ circumferentiæ datæque perpendiculari rectam datæ magnitudinis interjicere, quæ ad angulum ſemicirculi vergat.

Vide

Vide Tab. I. fig. 3, 4, 5, 6, 7. Semicirculus ACB, cujus baſis AB, magnitudine & poſitione datus eſto. Ad Baſin AB perpendicularis DE, poſitione detur. Circumferentiæ ACB, rectæque DE, interjectam puta CF datæ magnitudinis, quæ ad A vergat.

Solutio Hugonis D'Omerique. (Analyſis Geomet. Lib. I. prop. 38.)

Caſus 1. Rectâ DE datam circumferentiam contingente, ſive punctis B, D, coeuntibus. Vide fig. 3.

Jungatur DC. Propter angulos ABF, ACB rectos, rectangulum $\overline{FA \times AC} = AB^2$. Datum igitur $\overline{FA \times AC}$. Data vero FC magnitudine. Data igitur & AC magnitudine: (per 84. dat.) ergo & poſitione, (per 4. 1. elem.) propter punctum A datum, datamque circumferentiam ACB.

Compositio.

Detur recta GH. Rectæ GH applicetur rectangulum, quadrato ex AB æquale, excedens quadrato. Exceſſûs latitudo eſto HK. In ſemicirculo ACB, ad punctum A, inſcribatur AC = HK. Producta AC donec rectæ DE occurrerit, problemati ſatisfacit.

Caſus 2. Rectâ ad baſin perpendiculari, circumferentiæ datæ nullibi occurrente, & rectâ datâ perpendiculari & convexæ circumferentiæ interjectâ. Vide fig. 4.

Junctâ ut prius BC, ſimilia ſunt triangula ACB, ADF, propter angulos ad C, D, rectos, angulumque ad A utrisque communem. Ac proinde FA:AD = AB:AC, & rectangula $\overline{FA \times AC}$, $\overline{AD \times AB}$ æqualia. Datum igitur $\overline{FA \times AC}$. Data vero recta FC magnitudine. Data igi-

tur & AC magnitudine; (per 84. dat.) Quare & positione, ut in casu priori.

LIMES.

Propter FA majorem quam AD, & AC minorem quam AB, CF major est quam BD.

Omnium igitur CF, in hoc casu BD minima.

COMPOSITIO.

Eadem ac præcedentis, nisi quod vice quadrati ex AB, rectangulum $\overline{AB \times AD}$ usurpandum est.

Casus 3. Rectâ ad basin perpendiculari circumferentiæ datæ nullibi occurrente, & rectâ datâ cavæ circumferentiæ & perpendiculari interjectâ. (Vide fig. 5.)

Junctâ BC, reperientur ut prius, $FA \times AC$, $BA \times AD$ æqualia. Datum igitur $FA \times AC$. Sed FC magnitudine data. (per hyp.) Data igitur & AC magnitudine; (per 85. dat.) Quare & positione. (per 4. 1. elem.)

COMPOSITIO.

Detur recta KH. Ad datam rectam KH applicetur rectangulum $= \overline{DA \times AB}$, deficiens quadrato. Defectûs latitudo esto KG, vel GH. Circuli, centro A, intervallis KG vel GH descripti, punctum C unum atque alterum definient.

LIMITES.

Vides requiri, ut datum $DA \times AB$ rectangulum datæ KH applicetur deficiens quadrato; quod ut fieri possit necesse est $DA \times AB$ majus non sit quam $\frac{1}{4} KH^2$. Neque igitur KH^2 minus, quam $4\,DA \times AB$. Fiat igitur (vide Tab. III. fig. 1.) $AP = 4\,AD$. Et rectis, BA, AP, media proportionalis sumatur AM. Erit AM minima

nima omnium, quæ circumferentiæ BA, rectæque DE, interponi poterunt. Biſectâ vero AM in L, circulus centro A, intervallo AL deſcriptus, circumferentiæ AB ſi occurrerit, occurſu ſuo (N) poſitionem minimæ (NAO) definiet. Occurrent vero circumferentiæ, quoties AL minor fuerit quam AB. Sed propter $AL^2 = \frac{1}{4} AP \times AB = AD \times AB$, recta AL rectis AD, AB, media proportionalis eſt. Minor igitur erit quam AB, quoties AB major fuerit quam AD. Aliâs AL rectâ AB non erit minor. Quoties non minor fuerit, circulus centro A, intervallo AL deſcriptus, circumferentiæ AB non occurret,[a] omniumque CF ipſa BD erit minima. Cujuſmodi caſum exhibet Tab. III. fig. 2. & in hoc caſu data KH rectâ BD non debet eſſe minor. Segmentorum vero KG, GH, ſi KG majus ſit, propter $KG \times GH = DA \times AB$, & DA non minorem quam AB, KG major erit quam AB, atque GH minor. Circulus igitur centro A, intervallo GH deſcriptus, circumferentiæ AB occurret. Circulus vero, eodem centro, intervallo KG, non occurret. Problematis igitur in hoc caſu conſtructio unica.

Aſt cum circulus centro A, radio AL deſcriptus, circumferentiæ ANB occurrerit, circulus, centro A, intervallo utroque KG, GH, deſcriptus, circumferentiæ AB occurret, & gemina erit problematis conſtructio; ad utrasque partes minimæ, modo recta KH non minor quam AM, major non ſit quam BD, quæ ad ſuas partes minimæ, omnium utique CF in hujuſmodi caſibus maxima eſt. Quod ſi KH major ſit quam BD, circulus intervallo GH deſcriptus circumferentiæ AB ſolus occurret, & unica erit problematis conſtructio, ad partes minimæ a BD contrarias.

[a] Niſi forſan in ipſo B puncto, ſi AD, AB (ac proinde AL, AB) æquales ſint.

Vereor

Vereor, ne in hiſce brevior fuerim, quam ut Tyronibus perſpicua ſint. In Libro autem ſecundo, plurimi occurrunt limites ejuſdem generis ſed intricatiores, quorum præcipuos in eorum gratiam fuſe expoſui. Reliquo igitur opere perlecto ad hunc caſum redeant, quem interea noverint ad ſequentia nullatenus neceſſarium.

Caſus 4. Rectâ ad baſin perpendiculari datam circumferentiam ſecante, & rectâ datæ magnitudinis, perpendiculari & cavæ circumferentiæ interjectâ. Fig. 6. Analyſis eadem ac ſecundi.

Limes.

Propter AC minorem quam AB, & AF majorem quam AD, FC minor eſt quam DB.

Omnium igitur CF in hoc caſu, DB maxima.

Compositio.

Eadem ac caſûs 2^i^. niſi quod recta AC, rectæ GK (non HK) inſcribenda eſt æqualis.

Hunc caſum, ut & tertium, D'Omerique præteriit.

Caſus 5. Rectâ ad baſin perpendiculari datam circumferentiam ſecante, & rectâ datæ magnitudinis, perpendiculari & convexæ circumferentiæ interjectâ. Fig. 7. Analyſis eadem ac ſecundi. Compoſitio quoque eadem ac ſecundi, ſed abſque limite.

Scholion.

Problematis noviſſimi credo caſum quemque Apollonio problema fuiſſe, ut problemata libri primi univerſa, quæ nobis quinque ſunt, illi novem fuerint; quot ſcilicet

ſcilicet à Pappo numerata ſunt: qui expoſito Theoremate (nobis lemmate hujuſce libri tertio) quod limiti *noni* problematis definiendo utile prædicat,[a] alioque adjecto, quo ipſum limitem declarat, ad alia ſtatim tranſit, quæ problemata libri ſecundi unice ſpectant.

LEMMA I.

Quadrati ſive Rhombi latere producto, duæ rectæ æquales, eidem Angulo exteriori non ſubtenduntur, ad angulum oppoſitum vergentes, & major ſemper eſt, quæ cum diagonali angulum majorem facit.

Vid. fig. 8. Quadrati ſeu Rhombi ABCD latere AB producto, angulo exteriori HBC ſubtendantur rectæ duæ EF, GH, quæ ad angulum D utræque vergant. Dico EF, GH inæquales, & GH majorem eſſe, quæ cum diagonali DB angulum majorem facit. Angulus etenim ABD, (ſemiſſis nempe anguli ABC) acutus eſt. DBH igitur obtuſus. Obtuſior igitur DFH. Recta itaque DH, rectâ DF major. Sed propter BH, DC parallelas, eſt EF:FD =EB:BC, & GH:HD=GB:BC. BG vero major eſt quam BE. Ac proinde ratio BG ad BC ratione BE ad eandem BC major. GH igitur ad HD rationem habet majorem quam EF ad FD. Sed ratio GH ad HD minor quam GH ad FD. Ratio GH igitur ad FD ratione FE ad eandem FD multo major. GH igitur major erit quam EF. Inæquales igitur ſunt EF, GH, & GH major. Q. E. D.

[a] Prop. 73. Lib. 7. Quæ in Cod. Biblioth. Savil. (de quo plura alio loco dicam,) inſcripta eſt λημμα χρησιμον εις τον τυ Θ' διωρισμενου διορισμον. Διωρισμενου aut omnino redundat, aut corrupte ſcribitur pro προβληματος.

LEMMA

Lemma II.

Quadrati ſive Rhombi ABCD (fig. 8.) lateribus (AB, BC) circa angulum quemlibet (ABC) productis, ſi angulis exterioribus alternis (CBH, ABK) æquales rectæ (GH, LK) ſubtendantur, quæ ad oppoſitum angulum D utræque vergant, æquales erunt, BG, BL, æquales DG, DL, æquales DH, DK; denique BK, BH æquales.

Si BG, BL æquales eſſe negas, ſit utravis major: puta BG, & cape BE=BL. Anguli LBD, EBD, æquales ſunt (propter figuram ABCD æquilateram.) Sed & LB=BE, (per conſtruct.) & BD triangulis utrisque LBD, EBD, latus eſt commune. Anguli igitur BLD, BED æquales. Ac proinde BLK, BEF æquales. Æquales vero LBK, EBF. (per 15. 1. elem.) Et LB, BE æquales. (per conſtruct.) Æquales igitur LK, EF. (per 26. 1. elem.) Sed LK=GH. (per hyp.) Æquales igitur ſunt EF, GH. Quadrati igitur ſeu Rhombi ABCD, latere AB producto, angulo exteriori EBF, æquales rectæ EF, GH, ſubtenſæ ſunt ad angulum D vergentes. Quod fieri non poſſe lemmate primo oſtenſum eſt.

Non ſunt igitur inæquales BL, BG. Æquales igitur. Propter æquales vero BL, BG, latuſque BD triangulis duobus LBD, GBD commune, anguloſque LBD, GBD æquales, æqualia ſunt DL, DG latera: æquales quoque BDL, BDG anguli. Sed LK=GH. (per hyp.) Quare DK=DH. Propter DB igitur triangulis duobus BDK, BDH commune, anguloſque BDK, BDH, oſtenſos æquales, æquales erunt BK, BH. Q. E. D.

Cor. 1. Si æquales ſunt LK, GH, angulis exterioribus alternis ſubtenſæ, ad angulum oppoſitum D utræque vergentes, anguli BDK, BDH, æquales erunt.

Cor. 2.

Cor. 2. In circulo ſunt puncta quatuor K, L, G, H; cujus centrum in rectâ DB eſſe manifeſtum eſt. Dico in DB ultra B productâ. Biſecetur enim GH in N, & ducatur NO ad angulos cum GH rectos, quæ rectæ DB occurret. Occurſus ſito O. Erit O centrum circuli per L, G, H, K. Ducatur a B in rectam GH perpendicularis BM. Quadrata $BM^2+GM^2=GB^2$. Et $BM^2+MH^2=BH^2$. Et propter AD, BG parallelas, $AD^2:AH^2=BG^2:BH^2$. Sed propter æquales AD, AB, minus eſt AD^2 quam AH^2. Quare BG^2 minus eſt quam BH^2. Ac propterea BM^2+GM^2 minora quam BM^2+MH^2. Ablatoque utrinque BM^2, minus erit GM^2 quam MH^2, ac proinde MG minor quam MH. GM igitur ſemiſſe rectæ GH, ſeu rectâ GN, minor eſt. Adjectâ utrique DG, erit DM minor quam DN. Sed propter BM, ON parallelas, DO:DB=DN:DM. DB igitur minor eſt quam DO.[a] Punctum igitur O in DB erit ultra B productâ. Q. E. D.

Problema IV.

Quadrati ſive Rhombi, magnitudine & poſitione dati, latere quolibet producto, rectam datæ magnitudinis angulo exteriori ſubjicere, quæ ad figuræ angulum oppoſitum vergat. Vide Tab. I. fig. 9.

[a] Hæc ita ſe habent, quoties punctum M, punctis G, H interjaceat, ſive quoties perpendiculum BM intra triangulum BGH ſit. Si BM caderet extra, punctum M punctis DG intereſſet. Fieri enim nequit ut ſit M in GH ultra H, cum angulus DHB, acuto DBA ſemper minor, acutior ſemper ſit. Puncto vero M intra D, G, locato (vide fig. 8. N°. 2.) DM certe minor quam DG, & multo minor quam DN. Et propter analogiam DB:DO=DM:DN, DB minor quam DO. In omni igitur caſu, O erit in DB ultra B.

Quadrati ſive Rhombi ABCD, magnitudine & poſitione dati, latus AB producatur. Angulo autem CBF exteriori interjectam puta EF datæ magnitudinis, quæ ad D vergat. Producto latere CB, angulo HBA æqualem puta ſubjectam GH, quæ ad D quoque vergat. Et jungatur DB. Æquales ſunt, BG, BE, necnon DG, DE; DH, DF; BH, BF: puncta autem H, G, E, F in circulo, cujus centrum in BD productâ. (per lemma 2. & Cor. 2.) Scribatur iſte circulus, & in DB productâ eſto centrum ejus K. Junge KE, KG. Rectæ vero DK & circumferentiæ circuli occurſus eſto L. Propter K centrum circuli, æquales ſunt KG, KE. Æquales autem GB, BE; (per lemma 2.) et BK triangulis utriſque, GKB, EKB, latus commune. Anguli igitur BKE, BKG æquales. Arcus igitur GE in L biſectus eſt. Angulus igitur BKE, ſeu DKE, angulo BFE, ſeu DFB æqualis. Angulus autem BDE, triangulis utriſque KDE, BDF communis. Reliquus igitur DBF, reliquo DEK æqualis, & triangula ſimilia. Quare KE:ED=FB:BD. Propter BE, AD vero parallelas, ED:EF=AB:BF. Ex æquo igitur in analogiis perturbatis, KE:EF=AB:BD. Datæ autem AB, BD, dataque datarum ratio. Data igitur eſt ratio EK ad EF. EF vero magnitudine data. Data igitur & KE magnitudine. (per 2. dat.) Anguli autem KBF, DBE, æquales ſunt, cum uterque angulo ABD æqualis ſit. Æquales igitur KBE, DBF. Sed & æquales jam oſtenſi ſunt DBF, DEK. Æquales igitur KBE, DEK. Angulus autem DKE triangulis BKE, DKE communis. Reliqui igitur BEK, KDE æquales, & triangula ſimilia ſunt. Quare DK:KE=KE:KB. Ac proinde $\overline{DK \times KB} = KE^2$. Datum igitur magnitudine $\overline{DK \times KB}$. Recta vero DB magnitudine

tudine & positione data. Datum igitur est punctum K, (per 59. dat.) centrum scilicet circuli HGEF, & ostensa est KE magnitudine data. Circumferentia igitur HGEF data est. Sed & rectæ AB, CB positione datæ. Puncta igitur E, F necnon G, H data sunt. (per 25. dat.) Rectæ igitur EF, GH, tam positione quam magnitudine datæ.

COMPOSITIO.

Data sit magnitudine recta M, cujus æqualis angulo CBF subjicienda est, ad D vergendo. Ut BD ad BA, fiat M ad N. Rectæ DB applicetur rectangulum quadrato ex N æquale, ut quadrato excedat. Excessûs latitudo esto BK. Centro K, intervallo N, circulus scribatur, qui rectis CB, AB in punctis E, F, G, H occurrat. Juncta EF problemati satisfacit. Jungantur enim DE, KE. Propter $\overline{DK \times KB} = KE^2$, DK:KE=KE:KB. Similia igitur sunt triangula DKE, EBK, & anguli EBK, DEK æquales. Sed EBK=DBF. Anguli igitur DBF, DEK æquales. Jungatur KF. Propter $\overline{DK \times KB} = KF^2$, erit DK:KF=KF:KB. Similia igitur triangula DKF, BKF. Et anguli KBF, KFD, seu KFE, æquales. Sed (propter æquales KF, KE) angulus EFK angulo KEF æqualis. Anguli igitur KBF, KEF æquales. Duo igitur DBF, KBF, duobus DEK, FEK æquales. Ac proinde duo DEK, FEK æquales duobus rectis. Recta igitur est linea DEF.

Propter angulos GBL, LBE æquales (ejusdem scilicet GBE dimidios) æquales sunt in circulo rectæ BG, BE. (per 7. 3. elem.) Sed & KG=KE. Angulus igitur GKB angulo BKE æqualis, & arcus GL arcui LE. Æquales igitur sunt anguli DKE, DFB. Angulus autem BDE

triangulis utriſque KDE, FBD communis. Similia igitur triangula DEK, DBF. Unde (ut in analyſi) oſtendetur, EK:EF=AB:BD. Sed EK (=N):M=AB:BD. (per conſtruct.) EF igitur æqualis eſt datæ M, & ad D vergit, angulo EBF ſubjecta. Q. E. D.

Similiter, juncta GH problemati in angulo ABH ſatisfaciet.

Circulum vero centro K, intervallo N, deſcriptum rectis AB, BC ſemper occurrere manifeſtum eſt, & bis utrique. Propter $DK \times KB = N^2 = KL^2$, & DK majorem quam KB, erit KL major quam KB. Punctum vero L ad circumferentiam: punctum K centrum. Punctum igitur B ſemper intra circumferentiam. Rectæ igitur per B, AB, CB, circumferentiæ, bis ſingulæ, occurrunt. Expoſita igitur problematis conſtructio, quoad magnitudinem datæ rectæ, limite omni caret: quo & *poſſe rei* pariter carere manifeſtum eſt. Nam ſi rectam DF, circa punctum D immotum, gyrari intelligas, eâ lege, ut termino ſuo F rectam AB productam ſemper attingat, varia erit longitudo rectæ EF in variâ anguli BDF amplitudine. (per Lemma 1.) Angulo autem FDB evaneſcente, ut DF, DB congruant, evaneſcit EF; contra BDF angulo in angulum BDC aucto, ut congruant DF, DC, in infinitum EF extenſa eſt.

Lemma III.

In omni triangulo Iſoſcele, rectarum omnium, quæ per punctum Baſeos medium ductæ lateribus intercipiuntur, Baſis minima eſt. [Æquales vero ſunt, quæ æquales hinc inde angulos cum Baſi faciunt.] *E Pappo, exceptis, quæ uncis incluſa ſunt.* Vide Tab. I. fig. 10.

Trianguli

Trianguli iſoſcelis BAC, biſectâ baſi BC in D, per D ducatur utcunque EDF. Dico EF baſi BC majorem. Angulus enim DFC angulo DCA, ac proinde angulo DBA, minor eſt. Ab angulo igitur DBE auferatur DBG angulo DFC æqualis. Similia erunt triangula DFC, DGB. Ac proinde DF:DC=BD:DG. Obtuſus vero angulus DCF, infra baſim ſcilicet iſoſcelis trianguli. Major itaque DF quam DC, vel BD. Quatuor igitur rectarum DF, DC, DB, DG, maxima eſt DF, DG minima. Tota igitur FG (ſive DF, DG ſimul) totâ BC (ſeu BD, DC ſimul) major. (Propter analogiam ſupra oſtenſam DF:DC=BD:DG, & per 25. 5. elem.)

Ducatur vero alia HDK, quæ faciat angulum BDH angulo BDE æqualem. Dico HK=EF.

Propter angulos enim EDB, BDH æquales, æquales erunt EDB, KDC. Sed & EBD=KCD, & DB=DC. Quare KD=ED (per 26. 1. elem.)

Rurſum propter angulos EDB, BDH æquales, æquales erunt BDH, CDF. Sed & DCF=DBH, & DC=DB. Quare HD=DF. (per 26. 1. elem.) Tota igitur HK, toti EF æqualis.

Cor. 1. Quæ majorem facit angulum cum baſi, major eſt.

Cor. 2. Quadrati ſive Rhombi ABCD, ſi angulo CBA (vid. Tab. II. fig. 1.) æquales rectæ GH, EF, ſubjiciantur, per angulum oppoſitum (D) tranſeuntes, æquales erunt BF, BH; FG, HE; BG, BE.

Ducatur per D recta MN, ad angulos cum junctâ BD rectos, quæ Rhombi lateribus BA, BC in M, N occurrat. Iſoſceles eſt MBN triangulum, & baſis MN in D biſecta eſt. Rectæ vero per D, FE, GH, lateribus interceptæ, æquales.

æquales. Æquales igitur ſunt anguli MDF, HDN (per converſ. lemm. 3.) Sed DMF=DNH, & DM=DN. Ergo MF=HN, & proinde BF=BH. Pari ratione propter angulos MDG, NDE æquales, necnon DMG, DNG, & propter DM=DN, æquales ſunt MG, NE. Totæ igitur FG, HE, necnon totæ BG, BE æquales.

Cor. 3. Puncta quatuor F, G, H, E ſunt in circulo, cujus centrum eſſe in BD manifeſtum eſt. Dico in BD ultra D productâ. Biſecetur EF in O. Ducatur OK ad angulos cum FE rectos: producatur BD utcunque ad Q. Propter angulum QDN rectum, erit QDO acutus. Rectæ igitur DQ, OK occurrent, & occurſus erit in BD ultra D. Occurſus autem (qui ſit K) circuli per E, F, G, H, centrum erit.

Problema V.

Quadrati ſive Rhombi magnitudine & poſitione dati, angulo cuilibet rectam datæ magnitudinis ſubjicere, quæ per angulum figuræ oppoſitum tranſeat. Vid. Tab. II. fig. 1.

Quadrato ſive Rhombo ABCD magnitudine & poſitione dato, angulo ABC ſubjectam puta EF datæ magnitudinis, quæ per D tranſeat. Jungatur BD, & agatur MDN ad angulos cum BD rectos, quæ Quadrati ſeu Rhombi lateribus AB, BC in M, N occurrat. Fiat angulus NDH angulo FDM æqualis; et occurrat HD Quadrati ſeu Rhombi lateribus AB, BC in G, H, punctis. Æquales erunt EF, HG. (per Lemma 3.) Æquales igitur FG, EH; BG, BE; BF, BH. (per Lemma 3. Cor. 2.) & puncta G, F, H, E in circulo, cujus centrum in BD ultra D productâ (per Lemma 3. Cor. 3.) Scribatur

batur iſte circulus, & centrum ejus eſto K. Recta vero DB circuli circumferentiæ in L occurrat, & jungantur KE, KF. Propter BF=BH, & BD triangulis utriſque DBF, DBH latus commune, anguloſque FBD, HBD æquales, æquales erunt DF, DH. Sed & KF=KH, & KD triangulis utriſque FKD, HKD latus commune. Anguli igitur DKF, DKH, ac proinde arcus FL, LH æquales. Angulus igitur LKF, ſeu BKF, angulo FEH, ſeu DEB, æqualis. Angulus autem KBF angulo DBE æqualis. Angulus igitur BFK=BDE, & triangula BKF, BED ſimilia. Quare BF:KF=BD:ED. Et propter parallelas DC, BF erit CD:BF=ED:EF. Exæquo igitur in analogiis perturbatis, CD:KF=BD:EF. Et permutando, CD:BD=KF:EF. Datæ vero CD, BD, dataque datarum ratio. Data igitur ratio KF ad EF. Data vero EF magnitudine. (per hyp.) Ergo & KF. (per 2. dat.) Rurſum anguli KFB, EDB oſtenſi ſunt æquales. Æquales vero EDB, KDF. Æquales igitur BFK, KDF. Similia igitur ſunt triangula BFK, FDK. Quare BK:KF=KF:KD, & $\overline{BK \times KD} = KF^2$. Datum eſt igitur $\overline{BK \times KD}$. Data vero BD magnitudine & poſitione. Datum igitur punctum K (per 59. dat.) quod circuli GFHE centrum eſt. Et oſtenſa eſt KF magnitudine data. Circumferentia igitur GFHE, poſitione data eſt. Sed & rectæ BA, BC poſitione datæ. Puncta igitur F, G, E, H data ſunt. Rectæque FE, HG tam poſitione quam magnitudine datæ ſunt.

LIMES.

Ad compoſitionem requiritur, ut circulus centro K, intervallo quodam dato, KF, deſcriptus rectis BA, BC (productis ſaltem) occurrat. Demiſſis igitur a K in rectas

rectas BA, BC perpendicularibus KP, KR, neceſſe eſt ut KF, æqualium utrâque KP, KR non ſit minor. Eſto æqualis. In hoc caſu circulus centro K, intervallo KF deſcriptus, rectas BA, BC, in P, R punctis continget, neque alibi alterutri occurret. Puncta igitur, F, G; H, E, in hoc caſu, in puncta P, R coeunt. Circulus autem diametro KB deſcriptus (propter angulos ad P, R rectos) per puncta P, R, trajicit, & in eiſdem punctis rectæ MDN occurret. Aliter enim non eſſent æqualia $BK \times KD$, KP^2, quæ æqualia eſſe debent propter puncta, P, F coeuntia. Congruunt igitur puncta P, M necnon R, N. Sed congruere oſtenſum eſt, in caſu poſito, puncta F, G in P, & E, H in N. Congruunt igitur, in eodem caſu, F, G in M; E, H in N. Unde certo colligitur MN, minima omnium, quæ problemati ſatisfaciunt; quod vel aliter & ſynthetice ex Lemmate 3°. oſtendi poteſt. Recta vero MN diagonalis AC dupla eſt. Occurrant enim diagonales AC, DB in S. Quadrati ſeu Rhombi diagonales, ad angulos rectos, & bifariam, ſe mutuo ſecant. Parallelæ igitur AC, MN, & BD rectæ BS dupla. Sed BD:BS=MN:AC. MN igitur rectæ AC dupla.

Compositio.

Detur recta T. Si T dupla ſit rectæ AC, recta MDN per D, ad angulos cum BD rectos utrinque ducta, problemati ſatisfaciet. Si inæquales ſint T, & 2 AC, neceſſe eſt, ut T major ſit. Fiat ut BD ad CD, ita T ad V. Rectæ BD applicetur rectangulum quadrato ex V æquale, excedens quadrato. Exceſſûs latitudo eſto DK. Centro K intervallo V circulus deſcriptus rectis BA, BC occurret.

occurret. Occurſus ſunto F, G, E, H. Jungantur DE, DF. Dico EDF rectam, & problemati ſatisfacere.

Rectam eſſe EDF ſic oſtendo. Propter $\overline{BK \times KD} = V^2 = KF^2$, ſimilia ſunt triangula BFK, FDK, & anguli BFK, FDK æquales. Jungantur KE, KH. Propter angulos FBK, HBK æquales, æquales erunt in circulum incidentes BF, BH. Sed æquales KF, KH, & latus BK triangulis utriſque, BKF, BKH, commune. Æqualia igitur & ſimilia ſunt triangula KFB, KHB, & anguli KFB, KHB æquales. Sed oſtenſi ſunt æquales KDF, KFB. Quare angulus KDF angulo KHB æqualis. Sed propter KE = V, æqualia ſunt $\overline{BK \times KD}$, KE^2. Similia igitur triangula BKE, EKD, & anguli BEK, (ſeu HEK) & KDE æquales. Sed propter KH = KE, anguli KHE, KEH æquales ſunt. Anguli igitur KDE, KHE æquales. Duo igitur KDF, KDE duobus BHK, EHK æquales; æquales igitur duobus rectis. Recta igitur EDF. Rectam jam oſtenſam EDF, datæ T æqualem eſſe, ex conſtructione quivis colliget, qui analyſin perpenderit; ſimili enim ratiocinio oſtendetur, ac in compoſitione problematis noviſſimi æquales oſtenſæ ſunt M & EF.

Junctis vero GD, DH, recta quoque erit GDH, & problemati ſatisfaciet.

SCHOLION.

Problematum duorum ultimorum conſtructiones expoſitæ, fallor, ni Apollonii fuerint. Quartum certe Problema, circuli ope Apollonium abſolviſſe, qui cum per E tranſiret, (Tab. I. fig. 9.) rectæ angulo CBF ſubjiciendæ, in alio aliquo puncto, occurrebat, Lemma Pappi ſextum (ſeu Math. Collect. Lib. 7. Prop. 70) utut corruptum, haud obſcurè indicat. Lemma iſtud, prout in pulcherrimo codice MS Bibliothecæ Savilienſis [a] legitur, ſic ſe habet; ῥόμβϰ ὄντος τοῦ $\overline{\alpha\delta}$, οὗ διάμετρος ἡ $\overline{\varsigma\gamma\varepsilon}$ (Vid. Tab. II. fig. 2.) ἐὰν τῶν $\overline{\varsigma\varepsilon}$, $\overline{\varepsilon\gamma}$ μέσῃ ἀνάλογον ληφθῇ ἡ $\overline{\varepsilon\zeta}$, [b] ϰ῾ ϰέντρον [c] μὲν τῷ ε, διαςήματι δὲ τῷ $\overline{\varepsilon\zeta}$, ϰύϰλος γραφῇ ὁ $\overline{\zeta\eta\vartheta}$, ϰαὶ ἐϰβληθῇ ἡ $\overline{\lambda\gamma\eta}$, ἔςαι εὐθεῖα ἡ διὰ τῶν $\overline{\eta\varkappa\varsigma}$. Quæ utut ſint veriſſima, (ſi $\overline{\varepsilon\zeta}$ pro $\overline{\varepsilon\zeta}$, reſtituas, alioqui enim omnino falſa ſunt) problemati tamen de Rhombo, vel indagando vel componendo, quâ ratione inſervire potuerint, divinare nequeo. In conſtructiones autem, quas expoſui, neſcio an incidiſſem

[a] De PAPPI CODICE Eximio apud *OXONIENSES.*

Codicis hujuſce, ut et alterius, qui in eâdem bibliothecâ cuſtoditur, benignè mihi copiam fecit Viri Reverendi & egregii, Aſtronomi Savilienſis ornatiſſimi, Thomæ Hornſby humanitas et amicitia. Continet, præter Συναγωγων Pappi Libros poſt ſecundum omnes, inedita duo Theodoſii opuſcula, alterum περὶ οἰϰήσεων illud, cujus in Synopſi Merſennus meminit, alterum in partes duas diviſum περὶ ἡμερῶν ϰαὶ νυϰτῶν, & libellum Autolyci περὶ ϰινουμένης σφαίρας. Omnia pervenuſtè, Itali cujuſdam ſcribæ manu, chartis linteis exarata, literis minoribus, Roberti Stephani typis ſpecie parum abhorrentibus. Acceſſêre commata, ſpiritus, & vocum accentus. Diagrammata denique nitidiſſime ornata. Ævum equidem recentius præ ſe ferunt chartarum candor, atque color atramenti parum ſubflaveſcens. Ætatis ducentos quinquaginta, ut 'plurimum, numerarem annos. Codex pulcherrimus in ſcriniis Bibliothecæ, juſto certè diutius, inutiliter latuit. Inutiliter diu latiturum timendum eſſet, ni ſpem meliorem faceret eximia Virorum optimorum præli curatorum induſtria. Neque enim ignorant, quanto Mathematicos univerſos demerituri ſunt obſequio, ſi Græca Pappi, hactenus inedita, in lucem emiſerint.

[b] Pro $\overline{\beta\zeta}$, quæ eſt Codicis lectio, Commandinus ες reſtituit, ſuadente demonſtratione.

[c] Fortaſſe περὶ ϰέντρου. Habet codex alter Savil. ϰέντρῳ.

dum

(dum omni ſtudio id molirer, ut ſolutionem aliquo modo ex Theoremate iſto derivarem, quod corruptum ſcilicet vix ſuſpicari fas erat, quia verum intelligerem) niſi errantem Hugo d' Omerique in viam revocaſſet: quem propria utique analyſis in conſtructionem problematis 4ⁱ, a noſtrâ parum abludentem, jam olim deduxit. Facto nempe angulo DEK (vid. Tab. I. fig. 9.) angulo DBF æquali, triangula DBF, DEK, EBK ſimilia animadvertebat. Hinc colligit $FE:EK=BD:AB$, & $EK^2=\overline{DK\times KB}$. (Analyſ. Geomet. Lib. 1. Prop. 42.) Datâ igitur rectâ M, fieri jubet M ad N ut BD ad AB; fieri autem $DK\times KB=N^2$, & ex puncto K poni $KE=N$. Dein jungi DE, junctamque, donec AB productæ occurrat, (in F) produci. Ita recta DF problemati ſatisfaciet.

Perpenſâ celeberrimi Geometræ Analyſi & conſtructione, ſtatim agnovi, circulum centro K intervallo N deſcriptum (qui omnino deſcribendus erat, ut definiretur punctum E) per puncta ſchematis noſtri (Tab. I. fig. 9.) F, H, G tranſire: et tam punctum F quam E, quin & G, & H, per occurſus ejus cum rectis AB, CB, definita eſſe. Hinc protinus conjectari ſubiit, alium non fuiſſe, cujus ope Apollonius rem confecerat, & ſchema pro Pappi lemmate, in codice Savilienſi, ita corrigendum eſſe, ut literis, γ, ϵ tranſpoſitis, γ ad Rhombi angulum eſſet, ϵ punctum aliud in diagonali, extra angulum γ, deſignaret. (In Tab. II. fig. 2. ſchema codicis exhibui; in fig. 3. ſchema codicis quo dixi modo emendatum.) Opinionem augebat, quod $\overline{\beta\varkappa\eta}$ in ſchemate emendato, iiſdem prorſus verbis rectam oſtendi ac $\overline{\beta\varkappa\eta}$ in ſchemate codicis, re perpenſâ, reperi; quin & ipſius Pappi verbis, modo Pappi

fuerint, quæ in codice occurrunt, non aliter correcta, quam corrigi omnino debent, ut in proprio ſchemate ſenſum aliquem ſanum præ ſe ferant, & demonſtrationem conficiant: unum locum ſi exceperis, ubi literæ binas quaſdam rectas deſignantes omnino (quod inſigne eſt) mutandæ ſunt, ut rectas alias deſignent, ſi quis ſchema codicis retinere maluerit, cum immutari minime debeant, ut in emendato ſchemate argumentum rite procedat. Hæc ut palam fiant, demonſtrationem ſubjungere viſum eſt, prout ſenſus poſtulat, emendatam. (Vid. Tab. II. fig. 2. & 3.)

Ἐπεζεύχθωσαν γὰρ αἱ [a] λε. εκ. ϛκ. κη. [b] εη. ἐπεὶ οὖν ἴση ἐστὶν ἡ ὑπὸ λγζ γωνία τῇ ὑπὸ [c] ζγκ, καὶ ἑφ' ἑκάτερα τ[d] τῶ κύκλω διαμέτρω εἰσὶν, αἱ λγ, γκ, ἴσαι εἰσίν. λῆμμα γάρ. ἀλλὰ καὶ ἡ λε τῇ εκ ἴση ἐστίν. Γωνία ἄρα ἡ ὑπὸ γλε τῇ ὑπὸ γκε[d] ἴση ἐστὶν. Ἀλλὰ καὶ ἡ ὑπὸ γλε ἴση ἐστὶ τῇ ὑπὸ γηε. [e] Καὶ ἡ ὑπὸ γκε ἄρα ἴση ἐστὶ τῇ ὑπὸ γηε. [f] Ἐστὶ δὲ καὶ ἡ ὑπὸ γκε [ἴση] τῇ ὑπὸ γϛκ, [propter εϛ : εκ = εκ : εγ. per conſtruct.) καὶ ἡ ὑπὸ γϛκ ἄρα ἴση ἐστὶ τῇ ὑπὸ γηε. [g] Ἀλλὰ καὶ ἡ ὑπὸ ηγε [h] τῇ ὑπὸ ϛγκ ἴση ἐστί. (nempe cum εγη = λγϛ & λγϛ = κγϛ) λοιπὴ ἄρα ἡ ὑπὸ γεη λοιπῇ τῇ ὑπὸ γκϛ ἴση ἐστίν. Ἀλλὰ ἡ ὑπὸ γεη μετὰ τ[d] ὑπὸ γκη δυσὶν ὀρθαῖς ἴσαι εἰσίν. [Producta enim κγ circulo iterum in ν occurrat, & jungatur εν. Propter angulos νγϑ, ηγϑ æquales, æquales erunt γν, γη. Sed εν = εη & εγ triangulis utriſque εγν, εγη latus commune. Angulus igitur νεγ = ηεγ, ac proinde ϑεν = ϑεη, & Arcus νϑ, arcui ηϑ æqualis.

[a] Pro λε. εκ, ut ad Schema *Codicis* demonſtratio accommodetur, legenda ſunt γλ, γκ. *Lectio Codicis* ſchema emendatum potius quam proprium reſpicere videtur. Quod notatu certe dignum eſt, & ex ſequentibus manifeſtum fiet.

[b] εη ſupplevi. Exigunt enim quæ ſequuntur, ut εη juncta ſit.

[c] Sic reſtitui pro ζηγ, ſenſu poſtulante & ſuffragante codice altero.

[d] Literam ε ſupplevi.

[e] Sic reſtitui pro γηϑ.

[f] Sic reſtitui pro γκε. In altero codice leguntur ordine inverſo, κῆ ἡ ὑπὸ γηε ἄρα ἴση ἐστὶν τῇ ὑπὸ γκε.

[g] Sic reſtitui pre γκε, ſenſu & altero codice ſuadente.

[h] Sic reſtitui pro κγε, ſenſu & codice altero ſuadente.

Angulus

Angulus igitur γκη, ſeu νκη Angulo ϑεη æqualis. Duo igitur γεη, γκη duobus γεη, ηεϑ, ac proinde duobus rectis, æquales ſunt] καὶ ἡ ὑπὸ γκϛ, μετὰ τ ὑπὸ γκη γωνία (= γεη, γκη ſimul) δυσὶν ὀρθαῖς ἴσαι εἰσίν. Ὥϛε εὐθεῖά ἐϛιν ἡ διὰ τῶν ϛκη [1] σημείων.

Hæc qui cum ſchemate utroque, 2°. & 3°, contulerit, in utroque intelliget puncta β, κ, η, ad rectam eſſe: neque mihi temerè perſuaſiſſe videor, Apollonii conſtructionem, qualis a nobis expoſita eſt, fuiſſe, ab Hugonis indagine magis & demonſtratione quam operâ graphicâ diverſam.

Imperitiores vero, neve exiſtiment Hugonem D'Omerique (virum licet minimè contemnendum) prudentiorem vel feliciorem Apollonio, per compendia effeciſſe, quæ *Magnus* ille *Geometra* per Ambages; ideo ſcilicet quod nullâ circuli mentione factâ D'Omerique DE (Tab. I. fig. 9.) jungi jubet, & junctam producendo, rem totam abſolvit, de circuli cum rectis AB, BC, occurſibus alteris parum ſollicitus: cum nobis & Apollonio, junctas DE, EF rectam conficere, demonſtratio prolixior ſit. Circulus namque, tam in conſtructione Hugonis quam in noſtrâ, omnino adhibendus: neque enim abſque circulo punctum E definiendum eſt, neque aliter definitur, niſi quod circuli deſcripti & rectæ CB interſectio ſit. Circuli vero & rectæ CB gemina erit interſectio: gemina quoque circuli & rectæ AB. Tres autem præter E interſectiones ſi a punctis F, G, H, diverſæ eſſent, duplicem in angulis ſingulis HBA, FBC problematis conſtructionem analyſis ſuaderet, cum rei natura non niſi unam in angulo utroque capiat; ut lemmate primo ſatis oſtenſum eſt. Interſectiones iſtæ an a punctis, quæ

[1] Sic reſtitui, pro ϛκχ, codicis alterius auctoritate, ſuadente etiam ſenſu.

dixi

dixi, diverſæ ſint necne, D'Omerique Lectori ſuo explorandum reliquit: Apollonius, ipſâ problematis compoſitione, congruere oſtendit; unde fidem operæ ſuæ pleniorem adſtruit, quæ ſuſpecta eſſet, ſi plura præſtare videretur, quam *poſſe rei* admitteret, & in totò vel indaginis vel conſtructionis negotio nihil præteriit, quod inexplicatum vel mediocriter doctis moram injiceret. In limitibus vero problematum conſtituendis (ut id obiter in Tyronum gratiam moneam) ſive in Caſibus, quibus problema conſtrui non poſſit, diſtinguendis, & quot in quoque caſu conſtructiones capiat definiendo, præterquam quod opera ſemper Geometræ danda eſt, ut, quos analyſis promat limites, an *rei naturæ* inſint, exploret, & ineſſe oſtendat, quâ fieri poterit breviſſime, id etiam agendum, ut, quos *rei ineſſe* noverit, ex analyſi deducat. Analyſis etenim rite inſtituta veros cujuſque caſûs limites in lucem ſemper profert, &, ut plurimum, *rei naturæ* ineſſe ſimul evincit. Aliquando autem eveniet, ut limites rei naturæ ineſſe prius intellexeris, quam analyſin ullam inieris; quos, etſi aliunde forte melius & brevius oſtenderis, nihilominus ex analyſi etiam, quâ ratione derivare poſſis, ſedulo cogitare debes; eum ſcilicet in finem, ut certo tibi conſtet, an *conſtructionis tuæ* & *rei ipſius* limites iidem ſint. Ita enim optimo examini operam ſubjeceris. Quod ſi quando evenerit, ut alios *rei* limites, alios *conſtructionis tuæ* deprehenderis, certo te ſcias, aut in analyſi hallucinatum eſſe, aut conſtructionem ſaltem caſuum quorundam peculiarium non rei propoſitæ univerſam aſſecutum. Hæcce ni fallor problematum 4^i. & 5^i. exempla ſatis illuſtrant. In 4°. problemate, conſtructionem in omni angulo exteriori fore unicam, ex rei naturâ proſpectum habui, priuſquam problema

blema aggreſſus eſſem ; et in lemmate 1°. oſtendi. Conſtructionem pariter unicam ex analyſi derivari, ipſa compoſitionis ratio ſatis probat. In problemate 5°, rectarum angulo cuique interiori per angulum oppoſitum ſubjectarum aliquam fore minimam, delineato Rhombo ipſo intuitu agnovi: quænam vero minima eſſet, in lemmate 3°. oſtendi. Reſoluto poſtea problemate, an eandem minimam analyſis, quâ uſus ſum, ſuaderet, accuratè expendi. Idque ſemper Geometræ annitendum eſt, ut limites *rei & conſtructionis ſuæ* eoſdem eſſe comprobet; neque alio conſilio Apollonium crediderim compoſitionem problematis 4^i. & 5^i. prolixius concinnâſſe, quam aliâs opus eſſet. Quoties autem iſtiuſmodi limites ſint, ut Geometram paulo callidiorem ante inſtitutam analyſin latuerint, rei naturæ ineſſe, non aliâ ratione melius, quam ex ipſâ analyſi, plerumque colligendum eſt: et raro admodum, neque tamen nunquam eveniet, ut duplici illo labore, *ex poſſe rei*, & *ex ratione rem efficiendi*, limites petendi ſint.

Problematum 4^i. atque 5^i. Conſtructiones, a noſtris diverſas, ſed concinnas admodum, Heracliti & Hugenii, ſilentio præterire nefas eſſet.

Problema 4^m, in quadrato, hoc modo Heraclitus reſolutum dedit. (Vid. Tab. II. fig. 4.)

Quadrati magnitudine & poſitione dati ABCD, latere AB producto, angulo exteriori CBF rectam EF datæ magnitudinis ſubjectam puta, quæ ad angulum oppoſitum D vergat. Fiat angulus DFG rectus. Diametro DG ſemicirculus deſcribatur, per F nimirum tranſituturus. Quadrata $CB^2+EF^2=CG^2$. Datum vero CB^2, & EF^2. Datum igitur CG^2; ac proinde data CG magnitudine

nitudine. Sed & poſitione. Datumque punctum C. (per hyp.) Datum igitur punctum G. Datum vero & D. Recta igitur DG tam magnitudine quam poſitione data eſt. Circumferentia igitur DFG poſitione data. Data autem recta AB poſitione. Datum igitur punctum F. (per 25. dat.) Data igitur recta DF poſitione.

Compoſitio manifeſta eſt. Detur nempe recta M, cui æqualis facienda eſt EF. Fiat $CG^2 = M^2 + CB^2$. Diametro DG ſemicirculus deſcriptus, occurſu ſuo cum rectâ AB, punctum F definiet.

Theorema autem $CG^2 = EF^2 + CB^2$ Pappus ſic oſtendit. Ductâ ab F in rectam DG perpendiculari FH, triangula FHG, DCE ſimilia ſunt, propter utrumque triangulo DFG ſimile. Sed $DC = FH$. Ergo $FG = DE$. Sed $DG^2 = DF^2 + FG^2 = \overline{FD \times DE} + \overline{DF \times FE} + FG^2 = \overline{GD \times DC} + \overline{DF \times FE} + DE^2$. Sed $\overline{DF \times FE} + DE^2 = \overline{FD \times DE} + EF^2$. Ergo $DG^2 = \overline{GD \times DC} + \overline{FD \times DE} + EF^2$. Auferatur utrinque $GD \times DC$, erit $DG \times GC = \overline{FD \times DE} + EF^2$. Auferantur hinc inde æqualia $DC \times CG$, & $\overline{FD \times DE} - DC^2$, relinquentur $GC^2 = DC^2 + EF^2 = CB^2 + EF^2$. Q. E. D.

Simili computo, ſed prolixo admodum & minime obvio, in Rhombi caſu, facto angulo EFG angulo DAB æquali (vid. Tab. II. fig. 5.) junctiſque BG, BD, quadratum ex BG duobus ex EF & DB ſimul æquale Hugenius reperit. Datâ igitur EF magnitudine, data erit BG. Datum vero eſt punctum B, & recta DC poſitione datâ. Datum igitur punctum G. (per 31. & 25. dat.) & DG tam magnitudine quam poſitione data. Circuli igitur ſegmentum datâ baſi DG conſtitutum, quod angulum datum DFG capiat, poſitione datum eſt,

&

& circumferentiæ ſuæ cum rectâ AB poſitione datâ occurſu punctum F deſignat. Quintum etiam Problema, ſimili modo Hugenius reſolvit. (Vide Hugenii Illuſtrium quorundam Problematum Conſtructiones.)

Algebræ vero ope problema aggredientibus, calculum in æquationem dimenſionum quatuor (prout olim experti ſunt Carteſius, Newtonus aliique) ſemper exiturum, ex conſtructione noſtrâ Geometræ ſatis intelligent; quæ tamen, calculo peritè inſtituto, terminis imparibus caritura eſt.

Ad alia propero, legitimâ indagine poſt Apollonium nemini hactenus aggreſſa.

Lib. I.
Tab. I.

Fig. 1.

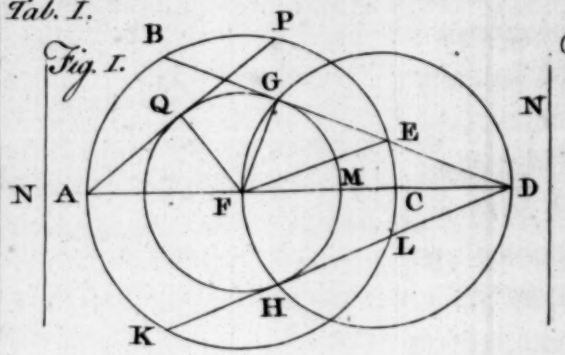


Fig. 2.

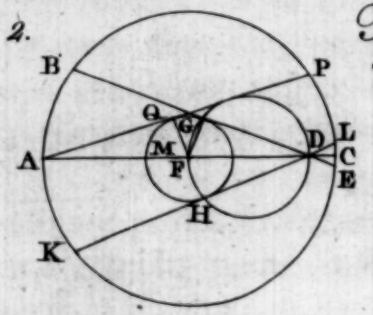


Fig. 3

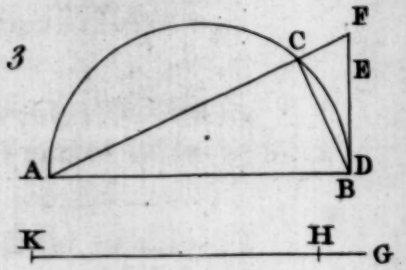


Fig. 4.

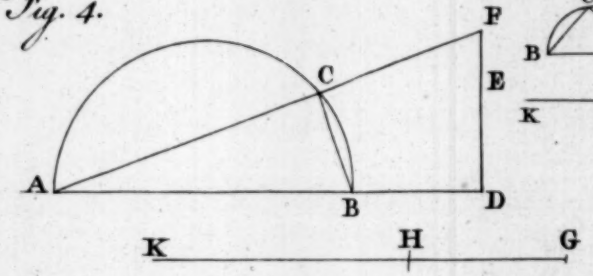


Fig. 5.

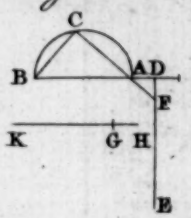


Fig 6

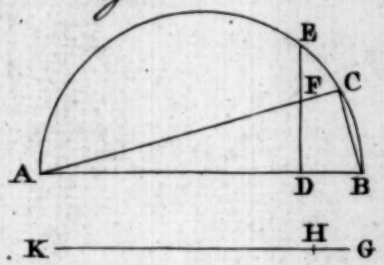


Fig. 7.

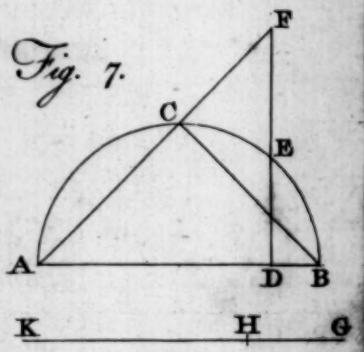


Fig. 8.
N.º 2.

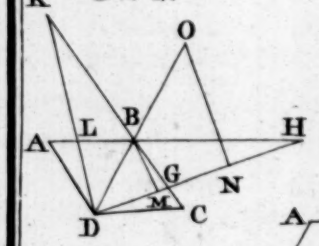
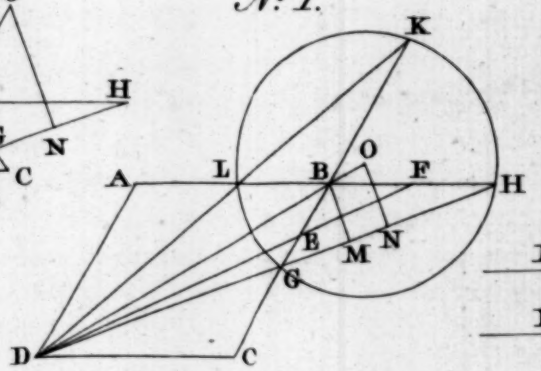


Fig. 8.
N.º 1.



M

N

Fig. 9.

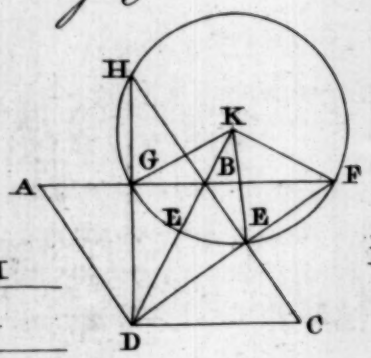
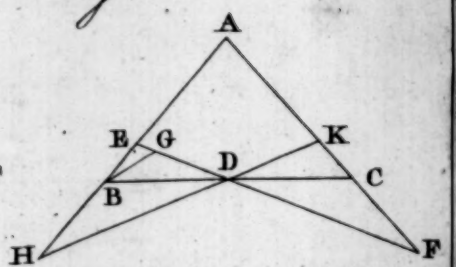


Fig. 10.



Lib. I.
Tab. II.

Fig. 4.

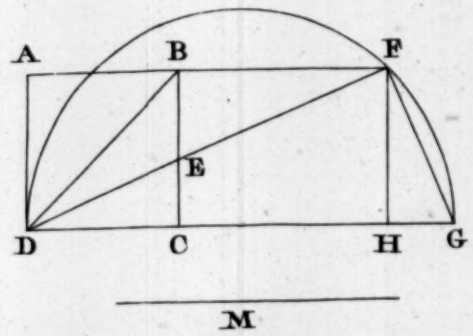


Fig. 5.

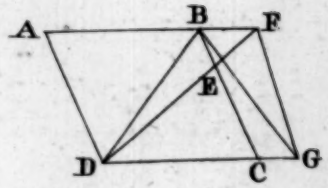


Fig. 3.

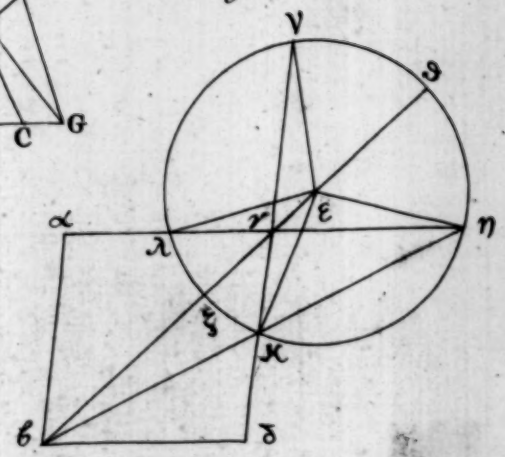


Fig. I.

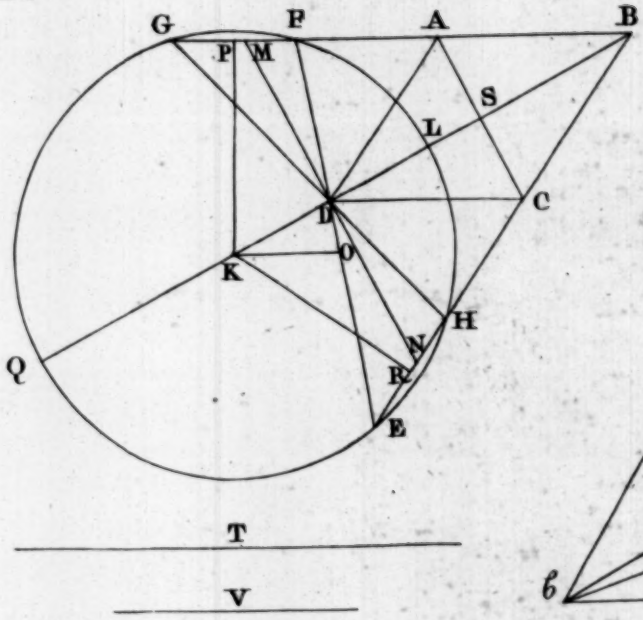


Fig. 2.

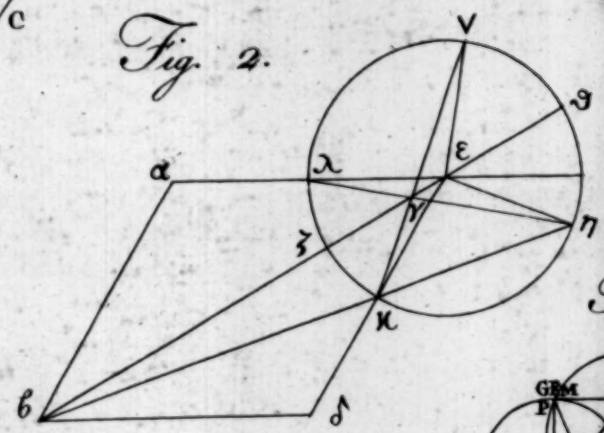
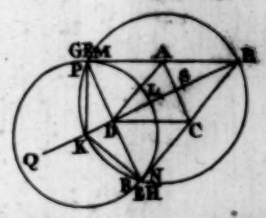


Fig. I. N.° 2.



Lib. I.
Tab. III.

Fig. 1.

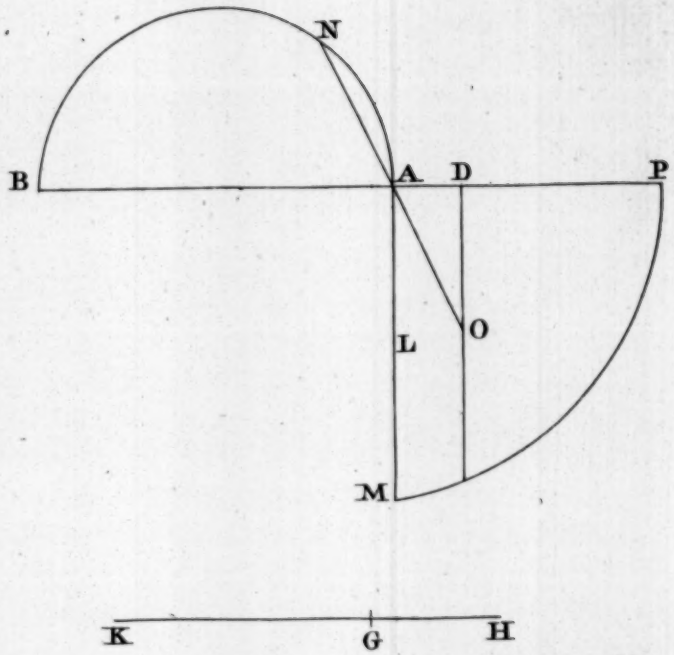
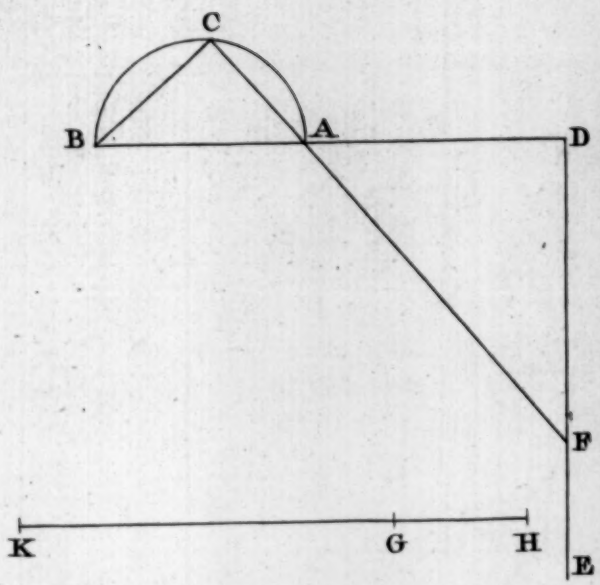


Fig. 2.



APOLLONII PERGÆI

INCLINATIONUM

LIBER SECUNDUS RESTITUTUS.

Lemma I. (é Pappo)

CIRCULOS concentricos AFB, CGD (Tab. I. fig. 1.) ſecet recta FGHK. Dico rectas FG, HK, nec non FH, GK æquales.

A centro circulorum communi E, in FK ducatur perpendicularis EL. Æquales erunt FL, LK. (propter circulum AFB.) Æquales etiam GL, LH. (propter circulum CGD.) Conſtat igitur propoſitum.

Cor. Circulum interiorem, recta MNubivis contingat, ut in P. Contingentis & exterioris circuli occurſus ſunto, M, N. Æquales erunt MP, PN.

Lemma II.

Rectæ magnitudine datæ (AB) (vid. Tab. I. fig. 2.) Terminus alteruter (A) detur. Terminus alter (B) eſto ad lineam cujuſcunque generis poſitione datam (DBC.) Dico rectam AB poſitione datam eſſe.

Centro A, ſemidiametro AB, ſcribatur circulus EBF. Data eſt poſitione circumferentia EBF. Sed & linea DBC poſitione data eſt. (per hyp.) Datum igitur punctum B. (per [a] 25. dat.) Ergo AB poſitione data eſt. (per 26. dat.)

PROBLEMA I.

Semicirculis duobus concentricis, in eâdem baſi, [b] poſitione datis, rectam datæ magnitudinis circumferentiis interponere, quæ ad punctum in baſi datum vergat.

INCLINATIO I.
Ad angulum ſemicirculi majoris.

Caſus bini erunt. Circumferentiæ enim rectam datam amplexæ, vel exterior concava, interior convexa, vel concavæ utræque erunt.

Semicirculis poſitione datis AFB, CGD, quorum AFB exterior eſt, centrum vero commune E, baſis ACDB, interjectam puta FG datæ magnitudinis, ad punctum B vergentem. (Vid. Tab. II. fig. 1. 2.) Recta autem BG circumferentiæ CGD occurat iterum in H. Propter circulos concentricos, rectæ BH, FG æquales ſunt. (per lemma 1.) Data eſt igitur magnitudine recta BH. Datum vero punctum B, dataque poſitione circumferentia CGD. Recta igitur BH poſitione data eſt. (per lemma 2.) Q. E. I.

[a] Propoſit. 25. Datorum de lineis quibuſcunque intelligenda eſt poſitione datis, licet doctiſſimus editor Gregorius rectas ſolas in ſchemate poſuerit.

[b] In ſequentibus, ſemicirculos, κοπυλοειδικως fateor, ſed brevitatis gratiâ, in *eâdem* baſi dico Conſtitutos, quorum baſes in rectâ ſunt.

LIMITES.

Limites.

In figurâ primâ, jungantur EF, EG, quarum EF occurrat circumferentiæ CHD in L. Æquales ſunt AC, FL. Recta autem FG, rectâ FL, ac proinde æquali AC, major. (per 8. 3. elem.) Ducatur a B, recta BN, circumferentiam CGD in N contingens. Producta autem BN circumferentiæ AFB in M occurrat. Rectangulum $\overline{GB \times BH} = BN^2$. Proportionales igitur ſunt GB, BN, BH; quarum GB major eſt quam BN. Ergo BN, ſive illi æqualis MN, major erit rectâ BH, ſive æquali FG.

Omnium igitur FG, in caſu primo, minima eſt AC, maxima MN.

In caſu ſecundo, (fig. 2.) BA major eſt quam BF. Et BD minor quam BG. Ergo AD rectâ FG neceſſario major.

Ducatur recta BNM, ut in caſu priori. BH major eſt quam BN. Sed BH = FG, & BN = NM. (per lemma 1. & Cor.) Ergo FG major erit quam NM.

In caſu igitur ſecundo, omnium FG maxima erit AD, NM vero minima.

In caſu autem utroque, neceſſe eſt ut ſemicirculi ad partes Baſeos eaſdem ſint.

Limitibus hiſce præfinitis, compoſitio utriûſque caſûs manifeſta eſt.

Inclinatio II.

Ad angulum ſemicirculi minoris.

Caſus ut prioris bini. (Vid. Tab. II. fig. 3. & 4.) Semicirculis poſitione datis AFB, CGD, quorum AFB major

major eſt, centrum vero commune E, baſis CABD, interjectam puta FG, datæ magnitudinis, ad B vergentem. Occurrat vero recta BG circumferentiæ CGD infra baſin continuatæ in H. Æquales erunt, ut in præcedentibus, FG, BH. Ergo BH magnitudine data eſt, ac proinde (per lemma 2.) poſitione. Q. E. J. Conſtructio autem eadem erit ac problematis noviſſimi, niſi quod punctum H, in circumferentiâ exteriori reperietur, cum in Inclinatione primâ in interiori fuerit.

Limites.

In figurâ tertiâ, oſtendetur AC (ut in primâ) minor quam FG. Ducatur per B, recta BN circulum AFB in B contingens. Circumferentiæ vero CGD in N occurrat. Angulus DBH acutus, minor eſt angulo DBN recto. BH igitur minor erit quam BN. Sed FG=BH. Ergo FG minor erit quam BN.

Omnium igitur FG, in caſu primo, minima eſt AG, maxima BN.

In figurâ quartâ, BH minor eſt quam BC. Sed BH=FG, & BC=AD. Ergo FG minor eſt quam AD.

Ducatur BN ut in figurâ tertiâ. Angulus acutus HBC minor recto CBN. HB igitur major eſt quam BN. Sed HB=FG. Ergo FG major quam BN.

Omnium igitur FG in caſu ſecundo, maxima erit AD, minima BN.

Porro, in caſu primo, neceſſe eſt ut ſemicirculi ad partes baſeos eaſdem ſint, in ſecundo ad contrarias. Compoſitionem, utpote manifeſtam, nil moror.

Inclinatio III.

Ad punctum in basi datum extra circumferentiam utramque.

Casus uti præcedentium bini. (Vid. Tab. II. fig. 5.6.) Semicirculis ALB, CMD positione datis, quorum ALB exterior est, centrum vero commune E, basis ACDB, interjectam puta rectam KH, datæ magnitudinis, ad datum punctum F vergentem. Esto autem punctum F in basi, extra circulos. Occurrat autem recta HK circumferentiis iterum in L, M. Jungatur EH, quæ circumferentiæ CMB occurrat in O, eidemque infra basim iterum in P. Erit igitur $\overline{PH \times HO} + EO^2 = EH^2$. (per 6^m. 2^i. elem.) Sed $\overline{PH \times HO} = \overline{KH \times HM}$. (per 36. 3. elem.) Ergo $\overline{KH \times HM} + EO^2 = EH^2$. Sed propter EH magnitudine datam, datum est EH^2. Datum igitur $\overline{KH \times HM} + EO^2$. Sed EO^2 datum, propter datam magnitudine EO. Datum igitur magnitudine $\overline{KH \times HM}$. Recta vero HK magnitudine data est. (per hyp.) Data igitur & HM magnitudine; (per 57. dat.) ac proinde illi æqualis KL, & tota HL. Datum vero punctum F, & circumferentia ALB positione data. Recta igitur FL positione data, per analysin prop. 1^a. Libri præcedentis. Q. E. I.

Limites.

In figurâ quintâ, ostendetur HK major quam AC, eodem modo, ac in fig. 1. & 3. ostensa est FG major quam AC.

Ducatur

Ducatur autem a puncto F recta FN, quæ circumferentiam CND contingat. Contactus autem esto N, rectæque FN & circumferentiæ AHB occursus sunto Q, R. Propter datum rectangulum KH × HM, spatio scilicet EH^2—EO^2, ut in analysi ostensum est, semper æquale, crescente KH, necessario imminuetur HM, ne vel manente HM, vel crescente utrâque, rectangulum quoque crescat. Maximâ igitur existente HK, minima erit HM, & KL, quæ rectæ HM semper æqualis est, simul minima. Sed, propter rectam FK rectâ FN semper majorem, rectamque FL rectâ FR semper minorem, omnium KL minima est NR. Ergo QN omnium HK maxima. Et ostensa est AC omnium HK minima.

In figurâ sextâ, ostendetur HK minor quam AD eodem modo, ac in fig. 2. ostensa est FG minor quam AD.

Ducatur autem a puncto F recta FRNQ, ut in casu priori. Propter rectam FH rectâ FQ majorem, rectamque FK rectâ FN minorem, HK major erit quam NQ.

In casu igitur secundo, omnium HK maxima est AD, & QN minima.

In casu autem utroque, necesse est ut semicirculi ad partes baseos easdem sint.

Inclinatio IV.

Ad punctum in basi datum inter circumferentias.

Casus sunt quatuor. Positio namque semicirculorum duplex; ad partes scilicet baseos aut easdem, ant contrarias. Et in utrâque positione casus bini, prout circumferentiæ rectam datam amplexæ, exterior concava, interior

interior convexa, vel concavæ utræque fuerint.

Vid. Fig. 7, 8, 9, 10.

Semicirculis AHB, CKD positione datis, quorum AHB major est, centrum commune E, basis ACDB, interjectam puta rectam HK datæ magnitudinis ad punctum datum F vergentem. Esto autem F in basi, inter circumferentias. Occurrat recta HK circulis iterum in L, M. Juncta EH circulo interiori in O, P occurrat. Rectangulum HM × HK, eodem modo ac in Inclinatione 3ª., ostendetur datum. Data autem HK magnitudine, (per hyp.) & illi æqualis LM. Data est igitur magnitudine & HM (per 57 Dat) & tota HL. Data vero circumferentia AHB, positione, datumque punctum F. Recta igitur HL positione data erit (per Analysin prop. 2ª. Libri præcedentis.) Q. E. I.

Limites.

In figura 7ª & 9ª, ostendetur AC minor quam HK, eodem modo ac, in fig. 1ª & 3ª, ostensa est AC minor quam FG.

Ducatur a puncto F recta circumferentiam CKD contingens. Contactus esto N. Rectæ autem FN & circuli exterioris occursus sunto Q, R. Existente HK maximâ, minima erit HM, id enim eodem modo ac in fig. 5. ostendetur. Sed, in fig. 7ª, propter rectam FH, rectâ FQ semper majorem, rectamque FM, rectâ FN semper minorem, omnium HM minima est QN. Eadem igitur QN omnium HK maxima. In figurâ autem 9ª, propter rectas HM, LK semper æquales, minimâ existente HM, LK simul minima erit. Omnium vero LK minima est NR, propter rectam FL rectâ FR semper majorem,

jorem, rectamque FK ſemper minorem rectâ FN. Ergo QN omnium HK maxima.

In fig. 8ª. & 10ª, oſtendetur HK, ſeu illi æqualis LM, minor quam AD, eodem modo ac, in fig 2ª, oſtenſa eſt FG minor quam AD.

Ducatur per F recta QNR ut in fig. 7ª. & 9ª. Et in fig. 8ª, eodem modo ac in 6ª, oſtendetur NQ minor quam HK. In figurâ autem 10ª, ſimili modo, oſtenditur LM major quam NR. Sed LM=HK, & NR=NQ. (per Lemma 2. & Cor.) Ergo HK major quam NQ.

In Caſu igitur hujuſce inclinationis primo & tertio, (fig. 7. & 9.) Omnium HK minima erit AC, maxima NQ.

In caſu autem ſecundo & quarto, (fig. 8. & 10.) omnium HK maxima AD, minima NQ.

INCLINATIO V.

Ad punctum in baſi datum intra circumferentiam utramque.

Caſus ſunt bini, ut Inclinationum trium priorum (Vid. Tab. II. fig. 11. & 12.) Semicirculis AHB, CKD, quorum AHB major eſt, centrum commune E, baſis ACDB, interjectam puta HK datæ magnitudinis ad datum punctum F vergentem. Eſto autem punctum F in baſi intra circumferentiam utramque, & occurrat HK circulis iterum in L, M. Eâdem prorſus analyſi, ac in præcedenti Inclinatione, ad Propoſitionem 2ᵐ Libri præcedentis res redigetur.

LIMITES.

In figurâ 11ª. oſtendetur HK major quam AC, eodem modo, ac in fig. 1ª, oſtenſa eſt FG major quam AC.

A puncto F ducatur FN ad baſin AB perpendicularis, quæ circulo minori in N, R, occurrat. Maximâ exiſtente HK, minima erit HM, ut in fig. 5ª oſtenſum eſt. Et KM neceſſario ſimul minima. Omnium vero KM minima NR, propter maximam a centro E diſtantiam. Omnium igitur HK, QN maxima.

In figurâ 12ª iiſdem conſtructis, propter DB omnium MH minimam, erit BC, ſive æqualis AD, omnium HK maxima. Et propter RQ omnium HM maximam, erit NQ omnium HK minima.

Porro in Caſu primo neceſſe eſt ſemicirculi ad partes baſeos eaſdem ſint, in ſecundo ad contrarias. Et in caſu utroque gemina eſt problematis conſtructio ad utraſque partes rectæ NQ.

Caſus problematis primi univerſos abſolvimus. Univerſi autem ſunt duodecim. Inclinationes nempe quinque; cujuſque Inclinationis, præter quartam, caſus bini; quartæ, quatuor.

Inclinationes tres poſtremas neſcio an Apollonius conſiderarit. Vix credo; certe Pappus in problemate generali non complexus eſt.

LEMMA III.

Duæ rectæ ſpatium datum in angulo dato complexæ, ſi altera datâ quâdam aucta vel imminuta, ad alteram eâdem datâ auctam vel imminutam, datam rationem habeat, magnitudine utræque datæ ſunt.

Caſus Primus. Rectis datum ſpatium complexis datâ utriſque auctis.

Rectæ AB, BC (Tab. I. fig. 3. N. 1 & 2.) datum ſpatium complexæ ſint in angulo dato. Rectangulum igitur $\overline{AB \times BC}$ datum erit [a]. Rectæ autem AB adjiciatur data AD, & rectæ BC eadem ſeu æqualis adjiciatur CE. BD autem ad BE datam habeat rationem. Dico, AB, BC ſingulas magnitudine datas eſſe. Propter datam enim rectarum DB, BE rationem, Rectangulorum $\overline{DB \times BC}$, $\overline{EB \times BC}$ ratio data eſt. Sed $\overline{DB \times BC} = \overline{AB \times BC} + \overline{DA \times BC}$, & $\overline{EB \times BC} = \overline{EC \times CB} + CB^2$. Spatium igitur $\overline{AB \times BC} + \overline{DA \times BC}$, ad ſpatium $\overline{EC \times CB} + CB^2$ datam rationem habet. Eandem habeat AB ad BF, & AD ad AG. Eandem igitur habebit $\overline{AB \times BC} + \overline{DA \times BC}$ ad $\overline{FB \times BC} + \overline{AG \times BC}$; ac proinde $\overline{FB \times BC} + \overline{AG \times BC} = \overline{EC \times CB} + CB^2$. Rectæ AG æqualis eſto EH. Erit igitur $\overline{FB \times BC} + \overline{EH \times BC} = \overline{EC \times CB} + CB^2$. Punctum vero H, erit vel inter puncta E, C, vel inter C, B. Fieri enim nequit ut ſit in CB ultra B, id quod infra oſtendam. Eſto primum punctum H inter E, C, ut in fig. 3. N. 1. Minor igitur erit EH quam EC, & æqualia utrinque auferendo, erit $\overline{FB \times BC} = \overline{HC \times CB} + CB^2$. Sed, propter rationem AB ad BF datam, rectangulorum $\overline{AB \times BC}$, $\overline{FB \times BC}$ ratio data erit, (per 1. 6. Elem.) & propter rectangulum $\overline{AB \times BC}$ datum, rectangulum $\overline{FB \times BC}$ datum erit. (per 2. dat.) Spatium igitur $\overline{HC \times CB} + CB^2$ æquale eſt ſpatio dato. Recta autem EC data. Et propter rectæ AD, ſive EC, ad rectam AG, ſive EH, rationem datam, data erit recta EH. (per 2. dat.)

[a] Nempe ſi duæ rectæ Spatium datum complexæ ſint in angulo dato, ſpatium eiſdem rectis in alio quovis dato angulo comprehenſum datum erit, quod monuiſſe ſatis eſto.

dat.) Ergo recta HC magnitudine data. Data igitur & CB (per 84. dat.) Spatium vero $\overline{AB \times BC}$ datum. Data igitur & AB. (per 57. dat.) Q. E. D.

Esto autem punctum H inter C, B, ut in fig. 3. N. 2. Et propter æqualia, ut supra ostensa sunt, $\overline{FB \times BC} + \overline{EH \times BC}$, & $\overline{EC \times BC} + CB^2$, æqualia itidem utrinque auferendo, relinquetur $\overline{FB \times BC} = BC^2 - \overline{CH \times CB}$. Sive $\overline{FB \times BC} = \overline{CB \times BH} = \overline{CH \times HB} + HB^2$. Datum vero $\overline{FB \times BC}$, id enim ut prius ostendetur. Data etiam ut prius EH, ac proinde data CH, datarum EC, EH differentia. Data igitur & BH (per 84. dat.) ac proinde tota BC, ac proinde AB.

Fieri autem nequit ut punctum H in CB reperiatur ultra B productâ. Si data enim ratio DB ad BE, majoris fuerit ad minus, propter DB : BE = AD : AG, AG, sive EH, necessario minor erit quam AD, vel EC. Et punctum H, ut in N. 1., punctis E, C, interjectum erit. Sin data ratio rectarum DB, BE, æqualium fuerit, æquales erunt EH, EC & puncta H, C congruent. Denique si ratio data DB ad BE minoris sit ad majus, AG major erit quam AD, ut in fig. 3. N. 2. EH igitur major quam EC. Sed propter rectam BD rectâ AD majorem, & DB : BE = AD : AG, erit AG sive EH minor quam EB. Punctum igitur H in hoc casu inter C, B.

Casus secundus. Rectis datum spatium complexis utrisque datâ imminutis. (Vid. Tab. I. fig. 4. N. 1. 2.)

Rectis AB, BC spatium datum in angulo dato complexis auferantur datæ & æquales AD, EC. DB vero ad BE rationem habeat datam. Dico AB, BC mag-

magnitudine datas. Rectangulorum enim $\overline{DB \times BC}$, $\overline{EB \times BC}$ ratio, ut in caſu priori, data. Spatii igitur $\overline{AB \times BC} - \overline{AD \times BC}$ ad $\overline{EB \times BC}$ ratio data. Eandem habeat AB ad BF, & AD ad AG. Eandem igitur habebit ſpatium $\overline{AB \times BC} - \overline{AD \times BC}$ ad ſpatium $\overline{BF \times BC} - \overline{AG \times BC}$. Ac proinde $\overline{BF \times BC} - \overline{AG \times BC} = \overline{EB \times BC}$. Fiat $EH = AG$. Erit igitur $\overline{BF \times BC} - \overline{EH \times BC} = \overline{EB \times BC}$. Et æqualia utrinque addendo $\overline{BF \times BC} = \overline{BH \times BC}$. In fig. 4. N. 1. $\overline{BH \times BC} = \overline{CH \times HB} + HB^2$. Spatium vero $\overline{BF \times BC}$, ut in priori caſu, datum oſtendetur. Datum igitur $\overline{CH \times HB} + HB^2$. Data vero oſtendetur CH, ut in caſu primo, N. 1. Data igitur & HB (per 84. dat) ac proinde tota BC, ac proinde & AB. In fig. 4. N. 2. $\overline{BH \times BC} = \overline{HC \times CB} + CB^2$. Datum igitur $\overline{HC \times CB} + BC^2$. Data autem HC, ut in Caſu 1°. N. 2. Data igitur & CB (per 84. dat.) ac proinde & AB. Q. E. D.

Si data ratio DB ad BE, majoris fuerit ad minus punctum H, punctis E,C interjectum erit, ut in fig. 4. N. 1.

At ſi data ratio DB ad BF minoris fuerit ad majus, punctum H reperietur in EC ultra C productâ, ut in fig. 4. N. 2.

Caſus Tertius. Alterâ ex rectis datum ſpatium complexis datâ quâdam auctâ, alterâ earundem eâdem imminuta. Vid. fig. 5.

Rectis AB, BC ſpatium datum in dato angulo complexis, alteri AB adjectam puta datam AD, alteri vero BC æqualem CE ablatam. Rectarum vero DB, BE ratio detur. Dico AB, BC magnitudine utraſque datas.

Rectan-

Rectangulorum namque $\overline{DB \times BC}$, $\overline{EB \times BC}$ rato data eſt. Id eſt, ſpatii $\overline{AB \times BC} + \overline{AD \times BC}$ ad $\overline{EB \times BC}$ ratio data eſt. Eadem eſto AB ad BF, & AD ad AG. Ita fiet $\overline{FB \times BC} + \overline{AG \times BC} = \overline{EB \times BC}$. Ponatur EH $=$AG. Propter DB: BE$=$AD: AG, & DB majorem quam AD, erit BE major quam AG, vel EH. Ac proinde propter $\overline{FB \times BC} + \overline{AG \times BC} = \overline{EB \times BC}$, æqualibus utrinque ablatis, reſtabunt $\overline{FB \times BC}$ & $\overline{HB \times BC}$ hinc inde æqualia. Datum vero $\overline{FB \times BC}$, ut in Caſibus præmiſſis. Datum igitur $\overline{HB \times BC}$, ſive $\overline{CH \times HB} + HB^2$. Data vero EH. (ut in caſibus præmiſſis) Data igitur CH, datarum EC, EH ſumma. Data igitur & HB (per 84. dat.) & tota BC, ac proinde AB. Q. E. D.

Lemma IV.

Duæ Rectæ ſpatium datum in Angulo dato Complexæ, ſi datis æqualibus ablatæ, relinquant aliquas in ratione datâ, magnitudine utræque datæ ſunt.

Detur AB & illi æqualis AC. Ablatæ AD, AE ſpatium datum in dato angulo complexæ, relinquant DB, EC in ratione datâ. (Tab. I. fig. 6. N. 1. 2. 3.) Dico AD, AE, magnitudine utraſque datas. Propter rectarum enim DB, EC rationem datam, rectangulorum $\overline{BD \times AE}$, $\overline{EC \times AE}$ ratio data erit. Sed rectangulum $\overline{BD \times AE} = \overline{AB \times AE} - \overline{AD \times AE}$. Spatii igitur $\overline{AB \times AE} - \overline{AD \times AE}$ ad ſpatium $\overline{EC \times EA}$ ratio data eſt. Eadem eſto AB ad AF, & AD ad AG. Ita fiet $\overline{AF \times AE} - \overline{AG \times AE} = \overline{CE \times EA} = \overline{AC \times AE} - AE^2$.

Vel

Vel æquales ſunt AF, AC, vel inæquales. Sunto primum æquales ut in fig. 6. N. 1. Æqualia igitur erunt $\overline{AG \times AE}$, & AE^2. Datum vero $\overline{AG \times AE}$, propter $\overline{AD \times AE}$ datum, datamque rectarum AD, AG rationem, quæ in hoc caſu æqualis eſt ad æquale. Datum igitur AE^2, ac proinde data AE, ac proinde & AD.

Si inæquales ſint, AF, AC, exceſſus primum eſto penes AF. (fig. 6. N. 2.) Eritque $\overline{AF - AC \times AE} + AE^2$, ſive (propter æquales AC, AB), $\overline{FB \times AE} + AE^2 = \overline{AG \times AE}$. Datum vero $\overline{AG \times AE}$. Datum igitur $\overline{FB \times AE} + AE^2$. Data autem AB. (per hyp.) Data igitur AF propter rationem AB ad AF datam. Data itaque & FB. Ergo et AE (per 84. dat.) ac proinde AD (per 57. dat.) Q. E. D.

Eſto autem inæqualium AF, AC exceſſus penes AC. (fig. 6. N. 3.) In hoc caſu erit $AE^2 + \overline{AF \times AE} - \overline{AC \times AE}$, ſive $AE^2 - \overline{FB \times AE} = \overline{AG \times AE}$. Ponatur EH = FB. propter AB : AF = BD : EC, & AB majorem quam BD, erit AF major quam EC. Sed AB = AC (per hyp.) ergo FB minor erit quam AE. Ergo EH minor quam EA, Erit igitur $AE^2 - \overline{FB \times AE}$, ſive $AE^2 - \overline{EH \times EA} = \overline{EA \times AH}$. Ergo $\overline{EA \times AH}$, ſive $\overline{EH \times HA} + HA^2 = \overline{AG \times AE}$. Datum igitur $\overline{EH \times HA} + HA^2$. Data vero EH, datæ FB æqualis. Data igitur & HA (per 84. dat.) & tota EA, ac proinde AD (per 57. dat.) Q. E. D.

Si data ratio BD ad EC, æqualis ad æquale fuerit æquales erunt AB, AF, ac proinde AC, AF æquales. Si ratio data minoris ad majus AF major erit quam AB, ac proinde quam AC. Si data ratio majoris ad minus fuerit, AF minor erit quam AB vel AC.

LEM-

LEMMA V.

Duæ rectæ ſpatium datum in angulo dato complexæ, ſi alterius earum & datæ cujuſdam ſumma vel differentia, ad duarum ſummam vel differentiam, rationem datam habeat, magnitudine utræque datæ ſunt.

Caſus Primus. Summæ ad Summam ratione datâ. (Vid. fig. 7. N. 1. 2. 3.)

Rectæ duæ AB, BC ſpatium datum complexæ ſint in angulo dato. Alteri AB, data AD adjiciatur, & detur ratio DB ad AC. Dico AB, BC, magnitudine utraſque datas. Data namque ratio rectangulorum $\overline{DB \times BC}$, $\overline{AC \times CB}$, ſive ſpatiorum $\overline{AB \times BC} + \overline{AD \times BC}$, $\overline{AB \times BC} + BC^2$. Eadem eſto AB ad BF, & AD ad AG. Ita fiet $\overline{BF \times BC} + \overline{AG \times BC} = \overline{AB \times BC} + BC^2$. Rectangula $\overline{BF \times BC}$, $\overline{AB \times BC}$ aut æqualia ſunt aut inæqualia. Sunto primum æqualia. (Fig. 7. N. 1.) Erunt igitur $\overline{AG \times BC}$, BC^2 æqualia. Ac proinde AG = BC. Data autem AG, propter rationem ejus, ad datam AD, datam, quæ in hoc caſu æqualis eſt ad æquale. Data igitur & BC. Ac proinde AB.

Sunto autem rectangula $\overline{BF \times BC}$, $\overline{AB \times BC}$ inæqualia, (vid. fig. 7. N. 2.) et exceſſus eſto penes $\overline{BF \times BC}$. Rectangulum $\overline{AG \times BC}$ neceſſario minus erit quam BC^2. Ac proinde AG minor quam BC. Fiat CH = AG. Punctum H punctis C, B, interjectum erit. Et propter $\overline{BF \times BC} + \overline{AG \times BC} = \overline{AB \times BC} + BC^2$, auferendo utrinque $\overline{AG + AB \times BC}$, reſtabunt $\overline{BF - AB \times BC}$, (ſive $\overline{FA \times BC}$) $\overline{BC \times BH}$ hinc inde æqualia.

Datum vero $\overline{AB \times BC}$. Et propter rationem AB ad BF, ſive $\overline{AB \times BC}$ ad $\overline{BF \times BC}$ datam, datum erit $\overline{BF \times BC}$. Datum igitur $\overline{FA \times BC}$, datorum nempe differentia. Datum igitur $\overline{BC \times BH}$, ſive $\overline{CH \times HB} + HB^2$. Data vero CH, datæ AG æqualis. Data igitur & BH (per 84. dat.) & tota BC, ac proinde AB. Q. E. D.

Inæqualium autem $\overline{FB \times BC}$, $\overline{AB \times BC}$ exceſſus eſto penes $\overline{AB \times BC}$. (Vid. fig. 7. N. 3.) In hoc caſu, propter $\overline{BF \times BC} + \overline{AG \times BC} = \overline{AB \times BC} + BC^2$, erit $\overline{AG \times BC}$ majus quam BC^2; ac proinde AG major quam BC. Ponatur $CH = AG$. Punctum H erit in CB ultra B. Propter $\overline{BF \times BC} + \overline{AG \times BC} = \overline{AB \times BC} + BC^2$, auferendo utrinque $\overline{BF \times BC} + BC^2$, reſtabunt $\overline{AG \times BC} - BC^2$, (ſive $\overline{CB \times BH}$) $\overline{AF \times BC}$ hinc inde æqualia. Datum vero $\overline{AF \times BC}$ ut ſupra oſtenſum eſt. Datum igitur $\overline{CB \times BH}$. Data vero CH, datæ AG æqualis. Data igitur & BH, (per 85. dat.) & reliqua BC ac proinde AB. Q. E. D.

Porro ſitus puncti H qualis futurus eſt facile proſpiciendus eſt. Scilicet ſi data ratio DB ad AC æqualis ad æquale fuerit, æquales erunt rectæ AB, BF, & ſpatia $\overline{AB \times BC}$, $\overline{BF \times BC}$ æqualia, & puncta H, B congruent, ut in fig. 7. N. 1.

Si data ratio DB ad AC minoris fuerit ad majus, major erit BF quam AB, & ſpatium $\overline{BF \times BC}$ ſpatio $\overline{AB \times BC}$ majus. Et punctum H punctis C, B interjectum erit, ut in fig. 7. N. 2.

Deni-

Denique ſi data ratio BD ad AC majoris ad minus fuerit, minor erit BF quam AB, & ſpatium $\overline{BF \times BC}$ ſpatio $\overline{AB \times BC}$ minus. Et puncti H in CB ultra B productâ locus erit.

Caſus ſecundus. Summæ ad Differentiam ratione datâ. (Vid. fig. 8.)

Rectæ AB, BC ſpatium datum complexæ ſint in angulo dato. Alteri ex iis BC data BD adjiciatur, et detur ratio DC ad AC. Dico AB, BC magnitudine utraſque datas. Data namque ratio rectangulorum $\overline{DC \times CB}$, $\overline{AC \times CB}$, ſive ſpatiorum $\overline{DB \times CB} + BC^2$, $\overline{AB \times CB} - CB^2$. Et invertendo & componendo, data erit ratio $\overline{AB \times CB} + \overline{DB \times CB}$ ad $\overline{DB \times BC} + BC^2$. Eadem eſto AB ad BF, & DB ad DG. Ita fiet $\overline{FB \times CB} + \overline{DG \times CB} = \overline{DB \times BC} + BC^2$. Ratio autem AD ad DC, quæ AB ad BF, & DB ad DG (perconſtruct.) ratio eſt, majoris eſt ad minus. DG igitur minor eſt quam BD, & rectangulum $\overline{DG \times CB}$ rectangulo $\overline{DB \times BC}$ minus eſt, ac proinde ſpatio $\overline{DB \times BC} + BC^2$ a fortiori minus. Æqualia igitur cum ſint $\overline{FB \times CB} + \overline{DG \times CB}$, $\overline{DB \times BC} + BC^2$, auferatur utrinque $\overline{DG \times CB}$, (idenim ſemper poteſt) et reſtabit $\overline{FB \times BC} = \overline{DB \times BC} - \overline{DG \times CB} + CB^2 = \overline{GB \times BC} + BC^2$. Datum vero, ut in ſuperioribus, $\overline{FB \times BC}$. Datum igitur $\overline{GB \times BC} + BC^2$. Data vero GB, datarum DB, DG, differentia. Data igitur & BC, (per 84. dat.) ac proinde AB (per 57. dat.) Q. E. D.

Caſus Tertius. Differentiæ ad Summam ratione datâ. (Vid. fig. 9. N. 1. 2.) Rectis AB BC ſpatium datum

in dato angulo complexis, alterius earum AB & datæ AD differentia esto BD. Ratio autem BD ad AC detur. Dico AB, AC, magnitudine utrasque datas. Data AD, vel minor est quam AB, vel major. Esto primum minor, ut in fig. 9. N. 1. Propter rationem BD ad AC datam, ratio rectangulorum $\overline{DB \times BC}$, $\overline{AC \times CB}$ data est. Data igitur ratio spatii $\overline{AB \times BC} - \overline{AD \times BC}$ ad $\overline{AB \times BC} + BC^2$. Eadem esto AB ad BF, & AD ad AG. Ita fiet $\overline{BF \times BC} - \overline{AG \times BC} = \overline{AB \times BC} + BC^2$. Et $\overline{BF \times BC} = \overline{AB \times BC} + \overline{AG \times BC} + BC^2$. Cum vero AB sit ad BF ut DB ad AC, BF major erit quam AB, & $\overline{BF \times BC}$ majus quam $\overline{AB \times BC}$. Æqualibus igitur $\overline{BF \times BC}$, $\overline{AB \times BC} + \overline{AG \times BC} + BC^2$ auferatur $\overline{AB \times BC}$. (potest enim.) Restabit $\overline{FA \times BC} = \overline{AG \times BC} + BC^2$. Datum vero $\overline{FA \times BC}$, datorum $\overline{AB \times BC}$, $\overline{FB \times BC}$, differentia. Datum igitur $\overline{AG \times BC} + BC^2$. Data vero AG, propter rationem, ad datam AD, datam. Data igitur & BC, (per 84. dat.) ac proinde & AB. Q. E. D.

Esto autem AD major quam AB. (Vid. fig. 9. N. 2.) Rectangulorum $\overline{DB \times BC}$, $\overline{AC \times CB}$, ut prius, ratio data. Data igitur ratio $\overline{AD \times BC} - \overline{AB \times BC}$ ad $\overline{AB \times BC} + BC^2$. Eadem esto AB ad BF & AD ad AG. Ita fiet $\overline{AG \times BC} - \overline{BF \times BC} = \overline{AB \times BC} + BC^2$. Et $\overline{AG \times BC} - BC^2 = \overline{AB \times BC} + \overline{BF \times BC}$. Datum igitur $\overline{AG \times BC} - BC^2$. Data vero AG. Data igitur & BC, (per 85. dat.) & proinde AB. (per 57. dat.)

Casus

Caſus quartus. Differentiæ ad Differentiam ratione datâ.

Rectis AB, BC, ſpatium datum in dato angulo complexis, alterius AB, & datæ AD, differentia eſto DB. (Vid. fig. 10. 11.) Ratio autem DB ad AC detur. Dico AB, BC, magnitudine utraſque datas. Data AD vel minor erit quam AB vel major. Eſto primum minor, ut in fig. 10. N. 1.

Data, ut in priori caſu, ratio rectangulorum $\overline{DB \times BC}$, $\overline{AC \times CB}$, ſive ſpatii $\overline{AB \times BC} - \overline{AD \times BC}$ ad ſpatium $\overline{AB \times BC} - BC^2$. Eadem eſto AB ad FB, & AD ad AG. Ita fiet $\overline{BF \times BC} - \overline{AG \times BC} = \overline{AB \times BC} - BC^2$. Rectangula $\overline{BF \times BC}$, $\overline{AB \times BC}$ vel æqualia ſunt, vel inæqualia. Si ſint æqualia, æqualia quoque erunt $\overline{AG \times BC}$ & BC^2. Ac proinde BC datæ AG æqualis.

Si inæqualia ſint, (Vid. fig. 10. N. 2.) & exceſſus ſit penes $\overline{BF \times BC}$, erit $\overline{BF \times BC} - \overline{AB \times BC} = \overline{AG \times BC} - BC^2$. Datum igitur $\overline{AG \times BC} - BC^2$. Data vero AG. Ergo & BC (per 85. dat.)

Si exceſſus fuerit penes $\overline{AB \times BC}$, (Vid. fig. 10. N. 3.) erit $BC^2 - \overline{AG \times BC} = \overline{AB \times BC} - \overline{BF \times BC}$. Datum igitur $BC^2 - \overline{AG \times BC}$. Data vero AG. Ergo & BC (per 84. dat.)

In omni igitur caſu, data BC, ac proinde & AB.

Eſto jam AD major quam AB. (Vid. fig. 11.) Oſtendetur igitur ratio $\overline{AD \times BC} - \overline{AB \times BC}$ ad $\overline{AB \times BC} - BC^2$ data. Eadem eſto AB ad BF, & AD ad AG. Ita fiet $\overline{AG \times BC} - \overline{FB \times BC} = \overline{AB \times BC} - BC^2$. Ac proinde $\overline{AG \times BC} + BC^2 = \overline{AB \times BC} + \overline{FB \times BC}$. Datum igitur $\overline{AG \times BC} + BC^2$.

Data

Data vero AG. Data igitur & BC, ac proinde AB. Q. E. D.

LEMMA VI.

(Vid. Tab. 4. fig. 5.) Detur rectangulum $\overline{AC \times CB}$, rectâ AB non datâ. Esto AC totius rectæ segmentum majus, BC minus. Dico, maximo existente AC, vel minimo existente CB, maximam fore AB. Propter datum enim rectangulum $\overline{AC \times CB}$, maximo existente AC, erit CB minimum. Contra minimo existente CB, maximum erit AC. Constat igitur propositum (per 25. 5 Elem.)

Cor. Bifariam secetur recta DE in F. Et æqualia sint DF^2, $\overline{AC \times CB}$. Erit DE minima omnium quæ ita secari poterunt ut segmenta rectangulum comprehendant dato $\overline{AC \times CB}$ æquale.

Cor. 2. Dato rectangulo $\overline{AC \times CB}$, crescente rectâ AB, crescet segmentum AC, & imminuetur CB.

LEMMA VII. (è Pappo, vid. Tab. 4. fig. 6.)

Detur magnitudine recta AB. Sint AC, CB inæquales, & minor esto AC. Dico, crescente rectangulo $\overline{AC \times CB}$, rectam AC crescere, CB imminui. Bifariam secetur AB in D. Erit $\overline{AC \times CB} + CD^2 = AD^2$. Datum igitur $\overline{AC \times CB} + CD^2$. Crescente igitur $\overline{AC \times CB}$ minuetur CD^2, ac proinde recta CD minuetur. Constat igitur propositum.

PROBLEMA II.

Semicirculis duobus in eâdem basi positione datis, rectam

rectam datæ magnitudinis circumferentiis ita interponere ut ad datum utriûſvis ſemicirculi angulum vergat.

Status Semicirculorum Primus.

Circulis contingentibus & altero alterum includente. Vid. Tab. III. fig. 1, 2, 3, 4, 5.

Inclinatio I.

Ad Contactum Circulorum communem.

Semicirculi duo AEB, ADC, in eâdem baſi ACB poſitione dati, contingant ſeſe in A. Circulus vero alter alterum includat. Exterior autem eſto AEB. Circumferentiis interjectam puta DE, datæ magnitudinis, ad contactum A vergentem. (Tab. III. fig. 1.) Propter contactum, ſegmenta AD, AE ſimilia erunt. Ac propterea, ut recta AC ad rectam AD, ita erit recta AB ad rectam AE. Ac proinde AC : AD = CB : DE. Sed propter datas AC, AB, data eſt CB. Ratio igitur CB ad DE data. (per 1. dat.) Data igitur ratio AC ad AD. Data vero AC magnitudine. Data igitur & AD magnitudine. Sed punctum A datum. Et circumferentia ADC poſitione data. Recta igitur AD poſitione data (per Lemma 2.) Q. E. D.

Limes.

Propter AC : AD = CB : DE, & AC ſemper majorem quam AD, CB ſemper major eſt quam DE.

Omnium igitur ED maxima eſt CB.

Ne-

Neceſſe eſt vero ſemicirculi ad baſeos partes eaſdem ſint.

Compoſitio manifeſta eſt.

Inclinatio II.

Ad Angulum ſemicirculi exterioris.

Caſus ſunt Bini, prout circumferentiæ datam rectam amplexæ, exterior concava, interior convexa, vel concavæ utræque fuerint. Vid. fig. 2, 3.

Semicirculi duo ADB, AEC in eâdem baſi ACB poſitione dati, in A contingant. Circulo autem altero alterum includente, exterior eſto ADB. Circumferentiis interjectam puta DE datæ magnitudinis ad B vergentem. Occurrat autem rectaDE circulo AEC iterum in F. Jungatur AD. Propter ſemicirculum ADB, angulus ADB rectus eſt. Eſto K centrum circuli AEC. Si a K, C, ad rectam DB, perpendiculares ducantur KL, CM, parallelæ erunt rectæ AD, KL, CM. Ac proinde AK : KC = DL : LM. Sed AK = KC. DL igitur = LM. Sed EL = LF, propter circulum AEC, & KL a centro perpendicularem. FM igitur æqualis eſt datæ DE. Sed propter AD, CM, parallelas, AB : BC = DB : BM. Data igitur Ratio DB ad BM. Datum vero rectangulum $\overline{EB \times BF}$, dato ſcilicet $\overline{AB \times BC}$ æquale. Duæ igitur rectæ EB, BF, datum rectangulum complexæ ſunt, et altera duarum, EB, datâ ED aucta, ad alteram, FB, eâdem datâ ED, ſeu æquali FM, imminutam, rationem datam habet. Rectæ igitur EB, BF, magnitudine utræque datæ ſunt. (per Lemma 3. Caſ. 3.) Data igitur BF magnitudine. Datum vero punctum

B, &

Lib. II.
Tab. I.

Fig. I.

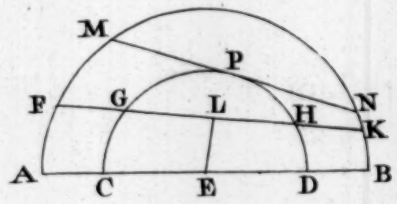


Fig. 3. N.o I.

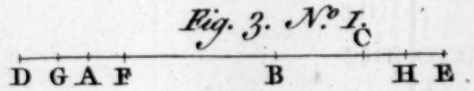


Fig. 3. N.o 2.

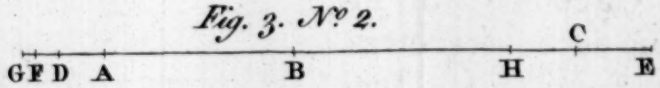


Fig. 4. N.o I.

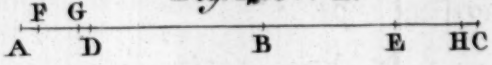


Fig. 4. N.o 2.

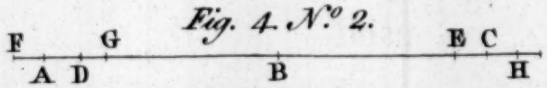


Fig. 5.

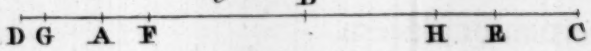


Fig. 6. N.o I.

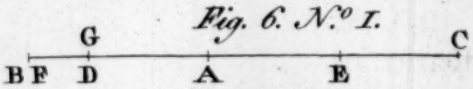


Fig. 6. N.o 2.

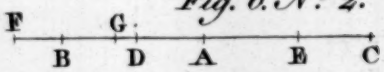


Fig. 6. N.o 3

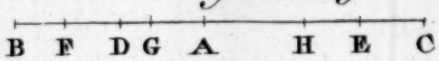


Fig. 2.

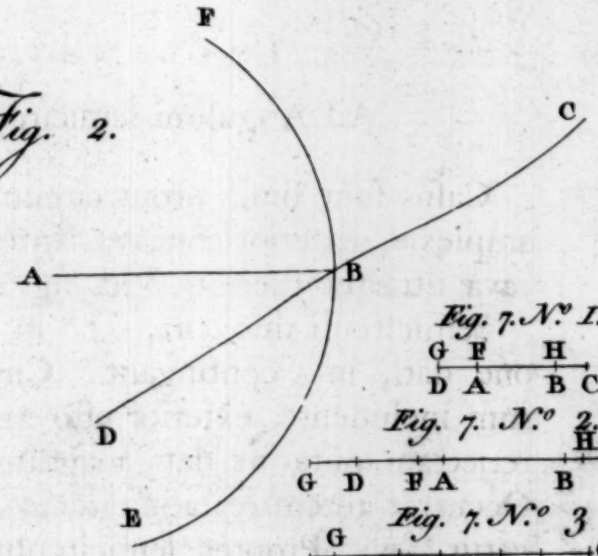


Fig. 7. N.o I.

Fig. 7. N.o 2.

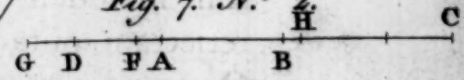


Fig. 7. N.o 3

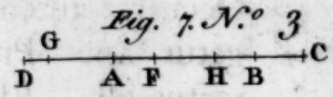


Fig. 8.

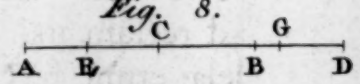


Fig. 9. N.o I.

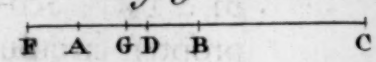


Fig. 9. N.o 2.

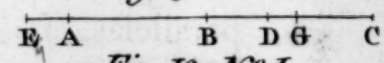
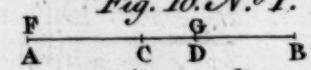


Fig. 10. N.o I.



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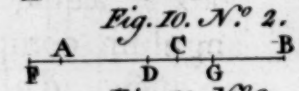


Fig. 10. N.o 3

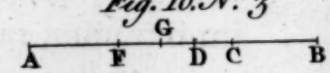
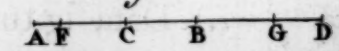


Fig. II.



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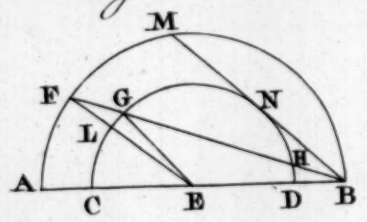
Tab. II.

Fig. 1.



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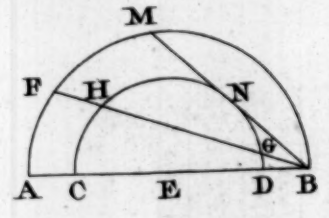


Fig. 3.

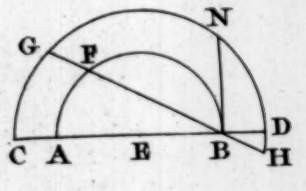


Fig. 4.

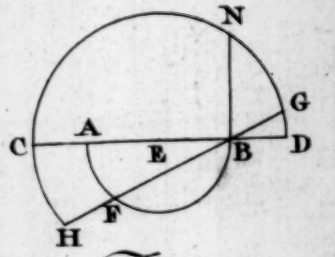
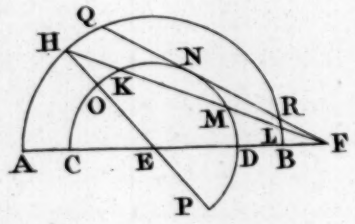


Fig. 5.



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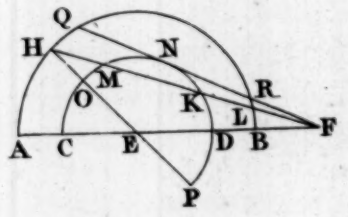

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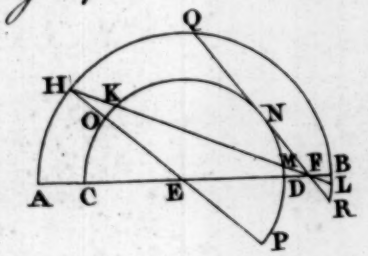


Fig. 8.

Fig. 9.

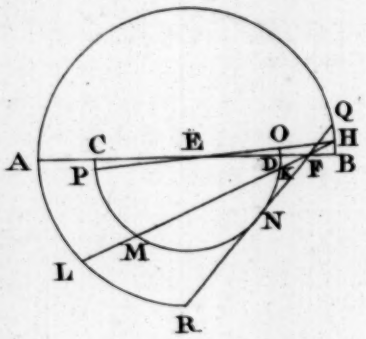
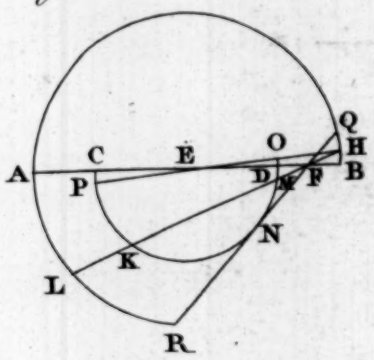


Fig. 10.



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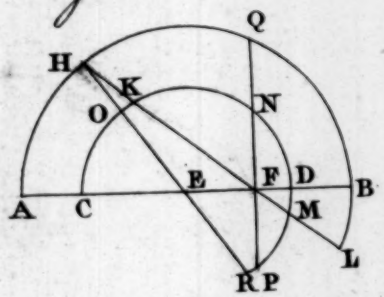

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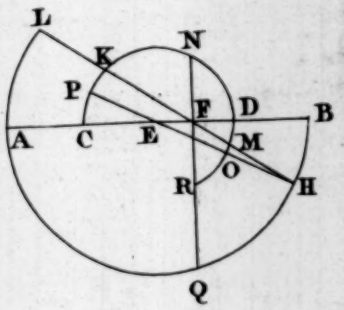


B, & circumferentia AEC poſitione data. Rectæ igitur BE, BF, poſitione datæ. (per Lemma 2.)

LIMITES.

In figurâ utrâque, ducatur BG circulum interiorem AEC contingens. Contactus eſto G. Rectæ vero BG & circumferentiæ exterioris occurſus alter eſto H.

In figurâ 2ª, jungantur KD, KH, KG, quarum KD circulo AEC in N occurrat. Angulus KGH rectus eſt (per 18. 3 Elem.) Ac proinde $HG^2+KG^2=HK^2$. Sed $\underline{DE \times DF}+NK^2=DK^2$. Eſto O centrum circuli ADB. Propter AB majorem quam AC, erit AO, ſemiſſis rectæ AB, major quam AK, ſemiſſis rectæ AC. Punctum igitur K inter puncta O, A. Ac propterea, HK major eſt quam DK, & HK^2 majus quam DK^2. Ac proinde HG^2+KG^2, majora quam $\underline{DE \times DF}+NK^2$. Et ablatis æqualibus KG^2, NK^2, majus erit HG^2 quam $\underline{DE \times DF}$. Sed propter BD majorem quam BH, & BF minorem quam BG, erit FD major quam HG. Ac proinde HG major erit quam DE.

In fig. 3ª. Propter BD majorem quam BH, & BE minorem quam BG, erit DE major quam HG.

In caſu igitur primo, omnium DE maxima eſt HG. In ſecundo, HG omnium DE minima, et AC ut ſatis patet maxima. Porro in utroque caſû, neceſſe eſt, ut ſemicirculi ad partes baſeos eaſdem ſint.

Limitibus hiſce præfinitis, Caſus utriuſque Compoſitio hujuſcemodi erit.

Data recta, cui DE æqualis eſſe debet, eſto QR.

Fiat ut AB ad BC, ita QR ad RS. Rectæ QS applicetur

cetur rectangulum quadrato ex BC æquale, ut quadrato excedat. Esto ST excessûs longitudo. Centro B, intervallo rectæ QT æquali, scribatur circulus, qui semicirculo AEC occurrat. Occursus esto F. Juncta BF, & producta, donec semicirculis iterum occurrerit, problemati satisfacit. Demonstrationem nihil moror, utpote ex præmissis satis obviam.

INCLINATIO III.

Ad Angulum Semicirculi interioris.

Casus, ut prioris, bini. (Vid. fig. 4, 5.)

Semicirculi duo positione dati, ADB, AEC in eâdem basi ABC, contingant in A. Circulo autem altero alterum includente, interior esto ADB. Circumferentiis interjectam puta rectam DE, datæ magnitudinis, ad B vergentem. Occurrat BE circulo AEC iterum in F. Rectangulum $EB \times BF$ datum est, dato scilicet $AB \times BC$ æquale. Junctâ AD, angulus ADB, propter semicirculum ADB, rectus erit. Esto K centrum circuli AEC. Ducatur a K in EF perpendicularis KL. Æquales erunt EL, LF. Fiat EM = BF. Æquales igitur erunt ML, LB. Et si a puncto M ducatur MH ad angulos, cum EF, rectos, & MH occurrat rectæ AC in H, erit (propter parallelas MH, LK) ML : LB = HK : KB. Æquales igitur HK, KB. Æquales vero AK, KC. Æquales igitur AH, BC. Data igitur est AH magnitudine. Sed est AB (per hyp.) magnitudine data. Data igitur (in fig. 4.) ratio AH ad AB. Sed (propter parallelas AD, HM) DM : DB = AH : AB. Ratio igitur DM ad DB data. Est autem DM rectæ BF, seu æqualis EM, & datæ ED differentia. DB vero ejusdem

dem ED & rectæ EB differentia eſt, et rectangulum EB × BF datum eſt. Rectæ igitur FB, BE magnitudine utræque datæ ſunt (per Lemma 3 Caſ. 2.)

In figura 5ª. propter datam rationem DM ad DB, data eſt ratio DM ad MB. Eſt vero DM datæ ED, & rectæ BF, ſeu æqualis FM, differentia; eſtque MB rectarum EB, BF differentia. Et rectangulum EB × BF datum eſt. Rectæ igitur FB, BE magnitudine utræque datæ ſunt. (per Lemma 5. Caſ. 4.)

Data igitur BF (in utrâque figurâ) magnitudine. Punctum vero B datum, dataque poſitione circumferentia AEC. Recta igitur BE poſitione data. (per Lemma. 2.) Q. E. D.

LIMITES.

In utrâque figurâ, ducatur BG circulum ADB in B contingens. Rectæ BG & circumferentiæ AEC occurſus eſto G.

In fig. 4ª. Eſto O centrum circuli ADB. Jungantur OG, OE, quarum OE circumferentiæ ADB in P occurrat. Propter angulum OBG rectum, $GB^2+BO^2=OG^2$. Sed $ED \times EB+OP^2=OE^2$. Sed OG major quam OE, (per 7. 3 Elem) ac proinde OG^2 majus quam OE^2. Ergo GB^2+BO^2 majora ſunt quam $ED \times EB+OP^2$. Et ablatis æqualibus BO^2, OP^2, reſtabit GB^2 majus quam $ED \times EB$. Major itaque eſt GB quam ut rectarum inæqualium EB, ED, quarum minor eſt ED, media proportionalis ſit; ac proinde minori illâ ED multo major.

In figura 5ª, propter angulum KLB rectum, KB major eſt quam KL; ac proinde EL major quam GB. ED autem major eſt quam EL. ED igitur major quam GB. EF vero non major quam AC. Eadem vero EF major quam ED. AC igitur major erit quam ED.

In caſu igitur primo, omnium ED maxima eſt BG.

In ſecundo, omnium ED minima eſt BG, AC maxima.

Porro neceſſe eſt in caſu primo, ſemicirculi ad partes baſeos ſint eaſdem, in 2° ad contrarias.

Limitibus hiſce præfinitis, caſûs utriuſque compoſitiones, ex Lemmatibus unde pendent, levi admodum negotio, Geometræ, pro ſe quiſque, derivabunt. Tyrones, ſi forte aqua illis hæreat, plenius rem intellecturi ſunt cum ad compoſitiones inclinationis 3ª., in Statu ſemicirculorum tertio devenerint: quæ adeo hiſce ſimiles et cognatæ ſunt, ut temporis jactura fieret, ſi ſingulis immorarer.

Status Semicirculorum Secundus.

Circulis contingentibus, & neutro alterum includente.

Inclinatio I.

Ad Contactum.

Semicirculi duo poſitione dati, ADB, AEC in eadem baſi BAC, in A contingant. Circulus vero neuter alterum includat. (Vid. fig. 6, 7, 8.)

Circumferentiis interjectam puta rectam DE, datæ mag-

magnitudinis, per contactum A tranſeuntem. (Vid. fig. 6.) Propter contactum ad A, ſimilia ſunt ſegmenta DA, EA. Ac proinde AB : AD=AC : AE. Ac proinde BC : DE=AB : AD. Data autem ratio BC ad DE, propter rectas BC, DE magnitudine datas. Data igitur ratio AB ad AD. Magnitudine autem data AB. Ergo & AD magnitudine data. Datum vero punctum A, & circumferentia BDA poſitione data. Data igitur AD poſitione. (per Lemma 2.)

LIMES.

AB major eſt quam AD, & AC major quam AE. Tota igitur BC, totâ DE major.

Omnium igitur DE maxima eſt BC.

Porro neceſſe eſt ut ſemicirculi ad baſeos partes ſint contrarias.

Compoſitio ſatis obvia eſt.

INCLINATIO II.

Ad alterum ſemicirculi utriuſvis angulum.

Caſus ſunt bini, prout circumferentiæ datam rectam amplexæ utræque convexæ, vel altera convexa, altera concava fuerint.

Circumferentiis interjectam puta DE, datæ magnitudinis, ad B vergentem. (Vid. fig. 7, 8.) Occurrat DE circumferentiæ AEC iterum in F. Et circuli AEC centrum eſto K. Jungatur AD, ductiſque a K, C, ad rectam BF, perpendicularibus KL, CM, æquales oſtendentur MF, ED eodem prorſus modo ac in fig. 2, 3. Rectæ vero

vero EB, BF rectangulum datum amplexæ ſunt, dato ſcilicet AB × BC æquale : dataque eſt ratio MB ad BD, datæ ſcilicet CB ad BA æqualis. Rectæ igitur BE, BF (per Lemma 3 Caſ. 3. ut in fig. 2, 3.) magnitudine utræque datæ ſunt. Ergo & poſitione (per Lemma 2.) Q. E. I.

LIMITES.

In utrâque figurâ 7, 8. ducatur a B, recta BG, quæ circulum AEC contingat. Contactus eſto G. Rectæ autem BG & circumferentiæ ADB, occurſus alter eſto H.

In fig[a] 7[a], BG major eſt quam BE, & BH minor quam BD.

Ergo GH major quam ED.

In fig[a] 8[a], jungantur AH, KG. Propter angulos ad G & H rectos, parallelæ ſunt KG, AH. Ac propterea, GH : HB = KA : AB = LD : DB. DB autem major eſt quam HB. LD igitur major quam GH. ED igitur multo major.

Iterum (propter parallelas MC, AD) MD : DB = CA : AB. Sed DB minor eſt quam AB. MD igitur minor quam CA, & a fortiori ED minor quam CA.

In caſu igitur primo, omnium DE maxima eſt GH.

In ſecundo, minima GH, AC maxima.

Porro in utroque caſu ſemicirculi neceſſe ad partes baſeos eaſdem ſint.

Compoſitio eadem fere ac in fig. 2. & 3. [a]

Semicirculorum contingentium caſus abſolvimus univerſos, qui ſunt octo. Contingentium nempe Status duplex, circulo altero incluſo vel neutro. In Statu pri-

[a] Id modo probe notetur BF in hiſce caſibus datâ augeri, BE imminui, cujus contrarium obtinuit in fig. 2. & 3.

mo

mo Inclinationes tres: primæ inclinationis Caſus unicus; cujuſque reliquarum duo. In Statu ſecundo Inclinationes duæ; primæ caſus unicus, ſecundæ duo.

Status Semicirculorum Tertius.

Circulis nullibi ſcilicet occurrentibus, altero alterum includente.

Concentricorum caſus omnes ſupra dedi expoſitos, in problemate primo. In ſequentibus igitur circulos nullibi occurrentes, incluſo altero ſemper pono non concentricos.

In concentricis, majoris ſemicirculi anguli ab angulis minoris, ſive a centro communi, æqualiter ſemper diſtant. In non concentricis, anguli exteriores, ab interioribus, & a minoris ſemicirculi centro, inæqualiter remoti ſunt.

Inclinatio I.

Ad Angulum ſemicirculi majoris, a minoris Centro remotiorem.

Caſus ſunt duo, prout circumferentiæ rectam datam amplexæ, exterior concava, interior convexa, vel concavæ utræque fuerint.

Semicirculi duo poſitione dati AEB, CFD in eâdem baſi ACDB nullibi occurrant. (Vid. Tab. IV. fig. 1, 2.) Circulo autem altero incluſo, major eſto AEB. Circuli interioris CFD centrum eſto H. Et rectarum inæqualium AH, BH, eſto BH major. Circumferentiis interjectam puta EF, datæ magnitudinis, ad B vergentem. Recta BF circulo CFD occurrat iterum in G. Rectangulum FB × BG datum eſt, dato ſcilicet

ſcilicet $\overline{CB \times BD}$ æquale. Ductâ autem HK ad BF perpendiculari, FK, KG æquales erunt. Fiat FM = BG. Æquales itaque erunt KM, KB. A puncto M ducatur ML ad angulos cum MB rectos. Propter ML, HK, itaque parallelas, MK : KB = LH : HB. Ac proinde LH = HB. Sed CH = HD. Æquales igitur LC, DB. Data eſt igitur LC magnitudine. Data igitur & LB (per 3. dat.) & datarum LB, BA ratio data eſt. Junctâ vero AE, angulus AEB, propter ſemicirculum AEB, rectus eſt. Rectæ igitur LM, AE parallelæ. Quare LB : BA = MB : BE. Ratio igitur MB ad BE data. Eſt vero BM, rectarum FB, BG (ſeu æqualis FM) ſumma, & BE, alterius BF & datæ EF, ſumma. Rectangulum vero $\overline{FB \times BG}$, ut ſupra oſtenſum eſt, datum. Rectæ igitur BF, BG magnitudine utræquæ datæ ſunt. (per Lemma 5. Caſ. 1. N. 2.) Data igitur BG magnitudine. Datum vero punctum B, & circumferentia BGC poſitione data eſt. Recta igitur BG poſitione data. (per Lemma 2.) Q. E. I.

LIMITES.

Eſto N centrum circuli exterioris. In utrâque figurâ, ducatur BP circulum interiorem contingens. Contactus ſit O. Circuli autem AEB, rectæque EO occurſus alter eſto P.

In fig^{a} 1^{a}, jungantur HO, HP, HE quarum HE circumferentiæ CFD in Q occurrat. Propter angulum HOP rectum, $HO^2 + OP^2 = HP^2$. Sed $\overline{EF \times EG} + HQ^2 = HE^2$. Majus autem HP^2 quam HE^2. (per 7. 3 Elem.) Majora igitur $OP^2 + OH^2$, quam $\overline{EF \times EG} + HQ^2$. Ablatis igitur HO^2, HQ^2 æqualibus, OP^2 majus eſt quam $\overline{EF \times EG}$. Recta vero

Lib. II.
Tab. III.

Fig. 1.

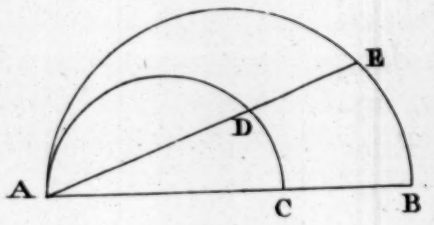


Fig. 2.

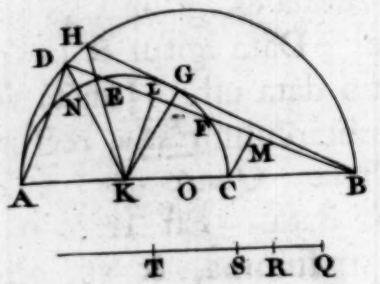


Fig. 3.

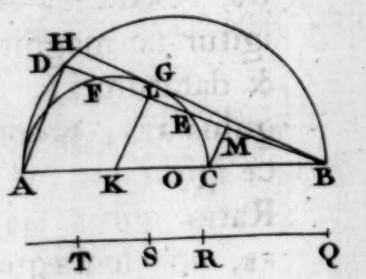


Fig. 4.

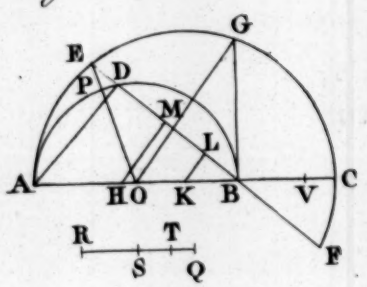


Fig. 5.

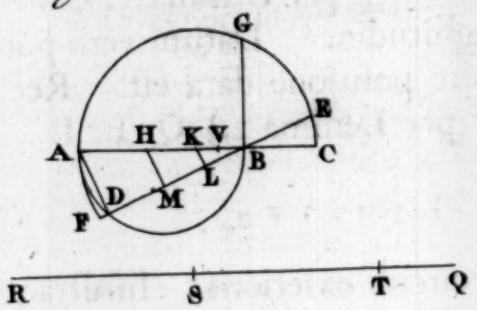


Fig. 6.

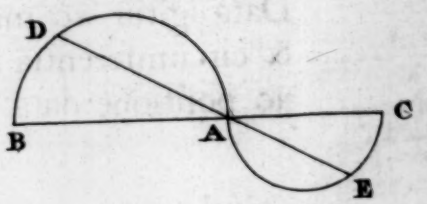


Fig. 7.

Fig. 8.

vero EG rectâ OP major, propter BE majorem quam BP, & BG minorem quam BO. Minor igitur erit EF, quam OP. Rurſus, propter HE majorem quam HA, & CH, HQ æquales, AC minor erit quam EQ, quæ & ipſa minor eſt quam EF.

In figurâ 2ª, BE major eſt quam BP, & BF minor quam BO. EF igitur major quam OP.

Rurſus BE minor quam BA, & BF major quam BD. Ergo EF minor quam AD.

In caſu igitur primo, omnium EF minima eſt AC, OP maxima.

In ſecundo, AD maxima, OP minima.

COMPOSITIO.

Data recta eſto RS. Fiat CL = DB. Ut AB ad BL fiat DB ad BV, & RS ad RT. Ad rectam RT applicetur rectangulum rectangulo $\overline{CB \times DV}$ æquale, excedens quadrato. Exceſſûs autem latitudo eſto TZ. Centro B, intervallo rectæ RZ æquali, ſcribatur circulus, qui circumferentiæ AFD in G occurrat. Juncta BG, & producta problemati ſatisfacit.

INCLINATIO II.

Ad angulum ſemicirculi majoris, Centro minoris propiorem. (Vid. Tab. IV. fig. 3, 4.)

Iiſdem ac in noviſſimâ inclinatione datis, circumferentiis AEB, AFD interjectam puta FE, datæ magnitudinis, ad A vergentem. Recta AF circumferentiæ CFD iterum in G occurrat. Datum igitur rectangulum $\overline{FA \times AG}$, dato $\overline{DA \times AC}$ æquale. Junctâ BE, angulus BEA rectus erit, propter ſemicirculum AEB. Ductâque HK ad rectam

FG perpendiculari, FK, KG æquales erunt. Factâ igitur FM=AG, æquales erunt MK, KA. Ductâque ML ad angulos cum MA rectos, propter ML, KH parallelas, LH, HA æquales erunt. Sed æquales HD, HC. Æquales igitur DL, CA. Data igitur LD magnitudine, totaque proinde LA. Et datarum BA, AL ratio data. Sed propter EB, LM, parallelas, BA : AL=EA : AM. Data igitur eſt ratio EA ad AM. Eſt vero EA, rectæ AF, & datæ EF ſumma. Et AM, rectarum AF, AG (ſeu æqualis FM) ſumma. Et rectangulum $\overline{AF \times AG}$ datum. Rectæ igitur AF, AG magnitudine utræque datæ ſunt (per Lemma 5. Caſ. 1. N. 3.) Data igitur AG magnitudine. Data igitur & poſitione (per Lemma 2.)

Compositio et Limites. Caſus 1.

Ducatur per A, recta AP, quæ circumferentiam CFD contingat. Contactus eſto O. Rectæ autem AO, & circumferentiæ AEB, occurſus eſto P. Eſto RS recta datæ magnitudinis, cujus æqualis circumferentiis AEB, OFD, interponenda eſt, ut ad A vergat. Fiat DL=AC. In datâ ratione BA ad AL, fiat AD ad DV, & RS ad RT. Ad rectam RT applicetur rectangulum rectangulo $\overline{AV \times AC}$ æquale, deficiens quadrato. Defectûs autem latitudo eſto RZ, vel ZT. Centro A intervallis ZR, vel ZT, circuli deſcripti, circumferentiæ C O ſi occurrant, punctum G unum atque alterum definient.

Vides vero ad compoſitionem requiri, ut ad datam rectam TR, rectangulum $\overline{AC \times AV}$ applicetur, deficiens quadrato. Quod ut fieri poſſit, neceſſe eſt ut quadratum ex dimidiâ TR minus non ſit rectangulo $\overline{AC \times AV}$. Ac proinde TR^2 minus eſſe non debet quam $4\overline{AC \times AV}$. Sed propter

ter analogiam BA : AL = RS : TR, $TR^2 = \frac{AL^2}{BA^2} RS^2$[a]. Ac proinde neceſſe eſt ut $\frac{AL^2}{BA^2} RS^2$ minus non ſit quam $\overline{4AC \times AV}$. Neque igitur RS^2 minus quam $\frac{BA^2}{AL^2} \overline{4AC \times AV}$. Cape igitur AY, rectæ AV quadruplam. (Vid. Tab. V. & VI.) Fiat BA : AL = AL : AQ. Fiat AQ : AB = AY : AX. Fiat AΠ = AC. Diametroque xΠ ſemicirculo ΠΩx conſtituto, ducatur AΩ, per A, ad angulos cum AX rectos, quæ ſemicirculo ΠΩx in Ω occurrat. Erit AΩ omnium minima, quæ ad A vergendo, in ullo ſemicirculorum ſitu, circumferentiis OB, PEB interponi poterunt[b]. Et ſi fiat AΩ ad AΦ, ut BA ad AL, erit AΦ omnium RT minima; et ſi AΦ bifariam ſecetur in Ψ, circulus centro A, intervallo AΨ deſcriptus circumferentiæ CGO ſi occurrat, occurſu ſuo poſitionem minimæ definiet. Quæ omnia evidentiora ſunt, quam ut ulteriori explicatione egeant.

Id vero diſquirendum eſt, an circulus centro A, intervallo AΨ deſcriptus circumferentiæ CGD ſemper occurſurus ſit; ſive quod eodem redit, an recta AΨ neque major unquam futura ſit quam AO, neque minor quam AC.

Sit, ſi fieri poteſt, AΨ = AO. Erit igitur & PO = AΩ. (ut ex analyſi præmiſſâ patet.) Et PO : 2 AO = AΩ : AΦ =

[a] Sive quadratum ex TR æquale eſt ſpatio ad quod quadratum ex RS rationem habet quam quadratum ex BA, ad quadratum ex AL. Magnitudinem enim, quæ tribus quibuſcunque A, B, C, quarta proportionalis eſt, ſic deſignare ſoleo, $\frac{B}{A}C$.

[b] Quæ vel brevius invenietur modo ſequente. Rectis AC, AV, capiatur media proportionalis quæ dicatur M. Fiat ut AL ad AB, ita M ad quartam, quæ dicatur N. Erit 2N minima omnium quæ ad A vergendo &c.

AB : AL. Ac proinde PO : AO = BA : AH. Junctis autem HO, PB, propter angulos ad O, P, rectos, rectaſque proinde HO, BP parallelas, BA : AH = PA : AO. Eſt igitur PO ad AO, ut PA ad AO. Ergo PO = PA, pars nimirum toti. Quod eſt abſurdum. Fieri igitur non poteſt, ut æquales ſint AΨ, AO.

Videamus an AΨ major eſſe poterit quam AO. Major autem ſit, ſi fieri poſſit. Fiat autem rectangulum $\overline{OA \times A\Theta} = A\Psi^2 = \overline{AC \times AV}$. Erit igitur AΘ major quam AΨ. Neceſſario igitur major quam AO. Ac proinde OΘ major erit quam 2AO. AΘ autem ea eſt ad quam PO datam rationem habet quam BA ad AL (ut ex analyſi præmiſſâ, & Lemmate 3°. Caſ. 1. N. 3. ſatis patet.) Habet igitur PO ad majorem quam 2AO rationem eam quam habet BA ad AL. PO igitur ad 2AO majorem habet rationem quam BA ad AL. Ac proinde PO ad AO majorem habet rationem quam BA ad AH: majorem igitur quam PA ad AO. Major igitur erit PO quam AP. Quod eſt abſurdum. AΨ igitur major non eſſe poterit quam AO. Sed oſtenſa eſt non æqualis. Semper igitur minor. Circulus igitur centro A, radio AΨ, deſcriptus circumferentiæ CGD ſemper occurſurus eſt, niſi forſan AΨ minor fuerit quam AG.

Ponantur vero æquales AΨ, AC. (Vid. Tab. V. fig. 1.) Eſt quadratum ex AΨ, æquale rectangulo $\overline{AC \times AV}$. Sed AΨ = AC. Ergo $AC^2 = \overline{AC \times AV}$. Puncta igitur C, V, congruunt. Ac proinde BA : AL = AD : DC. Permutando, BA : AD = AL : DC. Convertendo BA : BD = AL : 2AC. Ac proinde BA : BD = AH (ſeu $\frac{1}{2}$ AL) : AC. Et $\overline{BD \times AH} = \overline{AB \times AC}$. Id eſt $\overline{AB \times AC} = \frac{1}{2}\overline{DB \times AL} = \frac{1}{2}\overline{DB \times AD} + \overline{AC}$. Et ablato utrinque $\frac{1}{2}\overline{DB \times AC}$, erit

AB

$\overline{AB - \frac{1}{2}DB} \times AC = \frac{1}{2}\overline{BD \times AD}$. Ac proinde $AB - \frac{1}{2}DB : \frac{1}{2}DB = AD : AC$. Hinc dato D, facile invenietur circulus qui faciet DV, DC, nec non AC, AΨ æquales. Si talis ſit circulus COD (Tab. 5. fig. 1.) erit BD æqualis rectæ AΩ, & minima omnium quæ ad A vergendo circumferentiis OD, PEB interponi poterunt, OP autem maxima. Ducatur enim alia quævis AGFE. Fiat $EF : G\mu = BA : AL$ ($= BD : 2AC$.) Erit igitur $\overline{GA \times A\mu} = \overline{AV \times AC} = AC^2$, ut ex analyſi ſatis patet. Sed AG major quam AC. $A\mu$ igitur minor quam AC, totaque $G\mu$ major quam 2AC. (Lemma 6. Cor.) FE igitur major quam DB, propter analogiam $EF : G\mu = BD : 2AC$.

Porro maximâ exiſtente AG, maxima erit $G\mu$ (per Lemma 6.) Maxima proinde EF. (propter rationem EF ad $G\mu$ datam.)

Omnium igitur EF, minima erit DB, OP maxima. Q. E. D.

Ponantur autem inæquales AC, AΨ, & AΨ minor eſto. (Vid. Tab. V. fig. 2.) Minus igitur erit $A\Psi^2$ quam AC^2. Sed $A\Psi^2 = \overline{AC \times AV}$. Ac proinde $\overline{AC \times AV}$ minus eſt quam AC^2. Punctum igitur V, in hoc caſu, punctis C, A, interjacebit; et major erit DV quam DC. Ac proinde AD ad majorem quam DC rationem habet eam quam AB ad AL. Ratio igitur BA ad AL minor eſt quam AD ad DC. Permutando, ratio BA ad AD minor quam AL ad DC. Convertendo, ratio BA ad BD major quam AL ad 2AC. Major itaque quam AH ad AC. Rectangulum igitur $\overline{BA \times AC}$ rectangulo $\overline{BD \times AH}$ majus eſt. Majus itaque $\overline{BA \times AC}$ quam $\frac{1}{2}\overline{BD \times AL}$, ſive $\frac{1}{2}\overline{BD \times AD + AC}$. Ac proinde

AB—

$\overline{AB - \frac{1}{2}BD \times AC}$ majus quam $\overline{\frac{1}{2}BD \times AD}$. Et ratio $\overline{AB - \frac{1}{2}BD}$ ad $\frac{1}{2}$ BD major quam AD ad AC.

In hoc casu BD major erit quam AΩ, minima tamen omnium, quæ, ad A vergendo, circumferentiis OD, PEB interponi poterunt. OP autem earundem maxima.

Fiat enim BD ad CΣ, ut BA ad AL. Rectangulum $\overline{CA \times A\Sigma}$, rectangulo $\overline{AC \times AV}$, ac proinde quadrato ex AΨ æquale erit. (ut ex analysi & Lemm. 3. satis patet.) Recta autem AC major quam AΨ. (per hypothesin) Quare AΣ minor est quam AΨ, & tota CΣ major est quam AΦ. (per Lemma 6.) Propter analogias igitur BD : CΣ = BA : AL = AΩ : AΦ, erit BD major quam AΩ. Ducatur autem ab A, in circumferentias OD, PB, recta, AGFE utcunque. Et fiat EF : Gμ = BA : AL (= BD : CΣ). Rectangulum igitur $\overline{GA \times A\mu}$ = $\overline{AV \times AC}$ = $\overline{CA \times A\Sigma}$. Sed AG major quam AC. Aμ igitur minor quam AΣ, & tota Gμ, totâ CΣ major (per Lemma 6.) FE igitur (propter analogiam FE : Gμ = BD : CΣ) major quam BD. Porro maximâ existente AG, maxima Gμ, (per Lemma 6.) & maxima proinde EF. (propter rationem ÉF ad GM datam).

Omnium igitur FE minima est BD, & OP maxima.

Inæquales denique ponantur AΨ, AC, & major AΨ. (Vid. Tab. V. fig. 3. & Tab. VI.) Majus itaque $A\Psi^2$, sive $\overline{AC \times AV}$, quam AC^2. Punctum igitur V, in hoc casu, punctis C, D, interjectum erit. Et minor est DV quam DC. Simili igitur indagine ac in prioribus usi sumus, colligetur ratio $\overline{AB - \frac{1}{2}BD}$ ad $\frac{1}{2}$ BD minor quam AD ad AC.

In hoc vero casu circulus centro A, intervallo AΨ, descriptus circumferentiæ AO alicubi occurret. Occurrat in

in σ. Juncta Aσ & producta circumferentiis OD, PEB, in τ, υ, occurrat. Erit τυ æqualis AΩ, & omnium minima, quæ, ad A vergendo, circumferentiis OD, PEB interponi poterunt. Ducatur enim alia quævis AGFE in angulo primum υAP (ut in figura 3ª. Tab. V.) & fiat EF : Gμ = BA : AL (= AΩ : AΦ.) Erit igitur $\overline{GA \times A\mu} = \overline{AC \times AV} = A\Psi^2 = A\sigma^2$. Sed AG major quam Aσ, major itaque quam AΨ. Ac proinde Aμ minor quam AΨ, vel ΨΦ, totaque Gμ major quam AΦ. (per Lemma 6.) Propter analogiam igitur EF : Gμ = AΩ : AΦ. EF major eſt quam AΩ, vel ejus æqualis τυ.

Ducatur jam in alio angulo υAB recta quævis A ηθλ (Fig. eadem.) Fiat θλ : ην = BA : AL (= AΩ : AΦ.) Erit igitur $\overline{\eta A \times A\nu} = \overline{CA \times AV} = A\Psi^2 = A\sigma^2$. Sed ηA minor quam Aσ, vel AΨ. Aν igitur major quam AΨ vel ΨΦ, totaque ην major quam AΦ. (per Lemma 6.) Propter analogiam igitur λθ : ην = AΩ : AΦ, λθ major erit quam AΩ, vel ejus æqualis τυ.

Porro maximâ exiſtente AG, maxima erit Gμ (per Lemma 6.) & ſimul maxima EF. (propter rationem EF ad Gμ datam.) Omnium igitur quæ, ad A vergendo, circumferentiis Pυ, Oτ interponi poterunt, maxima eſt OP, & τυ oſtenſa eſt minima.

Minimâ autem exiſtente ηA, maxima erit ην (per Lemma 6.) & θλ ſimul maxima. (propter rationem θλ ad ην datam. Omnium igitur quæ, ad A vergendo, circumferentiis υB, τD interponi poterunt, maxima eſt BD, Et oſtenſa eſt τυ earundem minima.

Illud vero notatu non indignum eſt, & ad ſequentia utile: Si rectæ FE, Φε æquales ſint, ad utraſque partes mini-

minimæ, eſſe $\overline{AG+A\gamma}$ æqualem ei, ad quam utraque æqualium FE, $\Phi\epsilon$, datam proportionem habet quam BA ad AL. Quin & rectangulum $\overline{AG \times A\gamma}$ rectangulo $\overline{AC \times AV}$ æquale eſſe.

Rectis autem OP, DB vel æqualibus eſſe licet, vel inæqualibus, & inæqualium huic vel alteri eſſe majori.

Compositio Repetita.

Limitibus hiſce definitis compoſitionem accuratius exſequendam jam reſumo. (Vid. Tab. V. VI.) Secetur BD bifariam in I, & ſecetur AD in k, ut ſit, AD : Ak = AI : IB.

Eſto SR recta datæ magnitudinis, cujus æqualis circumferentiis OD, PB, interponi debet, ut ad A vergat. In datâ ratione BA ad AL, fiat DA ad DV. Si puncta K, C, congruant, (ut in Tab. 5. fig. 1.) congruent V, C, & data SR neque minor eſſe debet quam BD, neque major quam PO. (per ſuperius oſtenſa) In datâ autem ratione BA ad AL, fiat RS ad RT. Ad rectam RT applicetur rectangulum rectangulo $\overline{AC \times AV}$ (id eſt in hoc caſu AC^2) æquale, ut quadrato deficiat. (Poteſt enim, cum RT neceſſario major ſit quam $A\Phi$.) Defectûs Latitudo eſto RZ, vel ZT, quarum RZ minor ſit, ZT major, (atque idem ſemper intelligatur) Rectangulum $\overline{RZ \times ZT} = CA^2$. ZT igitur major quam CA, & RZ minor. Rectangulum vero $\overline{RZ \times ZT} = \overline{OA \times A\Theta}$. Sed propter analogias, SR : RT = BA : AL = PO : $O\Theta$, & PO majorem quam SR, erit $O\Theta$ major quam RT. Rectæ vero $O\Theta$, ſegmentorum OA, $A\Theta$, majus eſt OA. (Rectangulum namque $OA \times A\Theta = AC \times AV = A\Psi^2$, et oſtenſa eſt OA major

major quam AΨ. AΘ igitur utrâque AΨ, OA minor.) Major igitur OA quam TZ. (per Cor. 2. Lemma 6.) Circulus igitur centro A, intervallo TZ deſcriptus circumferentiæ CO occurret, cum TZ major ſit quam AC, & minor quam AO. Occurſus eſto G. Junctâ AG, & in circumferentias OD, PB, productâ, (quibus in F & E occurrat) erunt FE, SR æquales. Sed propter RZ minorem quam AC, circulus centro A intervallo RZ deſcriptus circumferentiæ CO nullibi occurſurus eſt. Unica igitur erit, in hoc caſu, rectæ AE poſitio.

Cadat autem punctum k inter puncta C, A, ut in Tab. V. fig. 2. Ratio igitur AB ad AL minor erit quam AD ad DC, (per ſuperius oſtenſa.) et punctum V punctis A, C, interjacebit. Atque iterum neceſſe erit, ut data RS neque minor ſit quam DB, neque major quam PO. (per ſuperius oſtenſa.) Iiſdem vero ac in fig. 1. conſtructis, factoque inſuper BD : CΣ = BA : AL, æqualia erunt $\overline{CA \times A\Sigma}$, $\overline{RZ \times ZT}$, propter utrumque rectangulo $\overline{AC \times AV}$ æquale. Sed propter analogias, RS : RT = BA : AL = BD : CΣ, rectamque RS rectâ BD majorem, erit RT major quam CΣ. Segmentorum vero CA, AΣ majus eſt AC (Rectangulum enim $\overline{C\Sigma \times A\Sigma} = A\Psi^2$, & AC major quam AΨ. Quare AΣ utrâque AΨ, AC minor.) TZ igitur major eſt quam AC, (per Lemma 6. Cor. 2.) Eadem vero ZT minor quam AO, (idenim ut prius oſtendetur.) Circulus igitur centro A radio TZ deſcriptus circumferentiæ CO (ut in priori figurâ) occurret. Occurrat in G. Juncta AG, & producta problemati ſatisfacit.

Recta vero RZ minor quam AΣ. (per Lemma 6. Cor. 2.) Minor igitur quam AC. Circulus igitur centro A radio RZ descriptus circumferentiæ CO nullibi occursurus est. Rectæ igitur AE, quæ problemati satisfacit, positio, ut prius, unica.

Cadat autem punctum k inter C & D, ut in Tab. V. fig. 3. & Tab. VI. fig. 1, 2. Major in hoc casu erit ratio AB ad AL, quam AD ad DC, & punctum V punctis D, C, interjacebit. (per superius ostensa.) Rectæ autem DB, OP, vel æquales sunt, vel non. Primo æquales sint. (Tab. VI. fig. 1.) Recta igitur RS utrâque OP, DB, non debet esse major. Inveniatur autem AΩ minima rectæ RS longitudo, & ducatur Aυ, cujus pars τυ rectæ AΩ æqualis sit : quorum utrumque quâ ratione efficiendum sit, satis in superioribus ostensum est. Tum facto ut prius SR : RT, nec non BD : CΣ = BA : AL, factoque etiam $\overline{TZ \times ZR} = \overline{AV \times AC}$, propter TR minorem quam O⊛, & majorem quam Aϕ, erit TZ, minor quam AO, & major quam AΨ,, vel Aσ. (per Cor. 2. Lemma 6.) Circulus igitur centro A intervallo TZ descriptus circumferentiæ σO occurret. Occursus esto γ. Propter TR vero majorem quam Aϕ, & minorem quam CΣ, erit ZR minor quam AΨ vel Aσ, & major quam AC. (per Lemma 6. Cor. 2. Segmentorum enim CA, AΣ, minus est AC, nempe cum rectangulum $\overline{CA \times A\Sigma}$ æquale sit $A\Psi^2$, & AC minor ponatur quam AΨ.) Circulus igitur centro A intervallo ZR descriptus circumferentiæ σC occurret. Occursus esto G. Junctæ Aγ, AG, Circumferentiis OD, PB, occurrant productæ in φ, ϵ,

F, E.

F, E. Erunt rectæ $\varphi\varepsilon$, FE, ad utraſque partes minimæ, rectæ RS ſingulæ æquales.

Inæquales jam ſint OP, BD, & BD major eſto. (Vid. Tab. V. fig. 3.) Inveniatur $A\Omega$, & ducatur $A\upsilon$, ut prius. Fiat BD : $C\Sigma$ = BA : AL = PO : OΘ. Propter DB majorem quam PO, erit $C\Sigma$ major quam OΘ.

Rectangula autem $\overline{OA \times A\Theta}$, $\overline{CA \times A\Sigma}$ æqualia, propter utrumque rectangulo $\overline{AC \times AV}$ æquale ; et ſegmentorum OA, AΘ minus eſt AΘ ; ſegmentorum vero CA, AΣ, minus eſt CA. Major igitur AΘ quam AC. (per Lemma 6. Cor. 2.) Eadem vero AΘ minor quam Aσ. Circulus igitur centro A intervallo AΘ deſcriptus circumferentiæ Cσ occurret. Occurſus eſto o. Juncta Ao circumferentiis τD, υB, producta in punctis ω, π, occurrat. Erit $\omega\pi$ = OP. Data RS neque minor eſſe debet quam $\tau\upsilon$, neque DB major. Quod ſi minor fuerit quam OP, facto ut prius RS : RT = BA : AL, factoque $\overline{RZ \times ZT} = \overline{AC \times AV}$, propter PO majorem quam RS, & propter analogiam RS : RT = PO : OΘ, major erit OΘ quam RT. Æqualia vero rectangula $\overline{OA \times A\Theta}$, $\overline{TZ \times ZR}$. AO igitur major quam ZT. Eâdem vero ZT minor erit AΨ, ſive Aσ, propter $A\Psi^2 = \overline{TZ \times ZR}$, et rectam TR rectâ A$\Phi$, vel 2A$\Psi$ majorem. (per Lemma 6. Cor. 2.) Circulus igitur centro A intervallo ZT deſcriptus circumferentiæ σO occurret. Occurſus eſto G.

Iterum propter $\overline{OA \times A\Theta} = \overline{RZ \times ZT}$, & propter AO majorem quam ZT, erit AΘ, ſive Ao minor quam RZ, Et propter rectangulum $\overline{RZ \times ZT} = A\sigma^2$, & ZT majorem quam A$\sigma$, erit RZ minor quam A$\sigma$. Circulus

igitur centro A intervallo RZ deſcriptus circumferentiæ σo occurret. Occurſus eſto γ.

Junctæ AG, Aγ, circumferentiis OD, PB, occurrant productæ, in F, E, φ, ε. Ita erunt rectæ FE, φε, ad utraſque partes minimæ, rectæ RS ſingulæ æqnales.

At ſi recta RS major fuerit quam OP, facto ut prius RS : RT = BA : AL, factoque $\overline{RZ \times ZT} = \overline{AC \times AV}$, propter RS minorem quam DB, erit RT minor quam CΣ. Erit igitur RZ major quam AC. (per Lemma 6. Cor. 2.) Sed propter analogiam RS : RT = BA |: AL = ωπ : Ao + AO, propterque RS majorem quam ωπ, erit RT major quam Ao + AO. Rectangula vero $\overline{RZ \times ZT}$, $\overline{Ao \times AO}$ æqualia, nempe cum utrumque rectangulo $\overline{AC \times AV}$ æquale ſit. Minor igitur RZ quam Ao. Circulus igitur centro A intervallo RZ deſcriptus circumferentiæ Co occurret. Occurſus eſto ν. Juncta Aν circumferentiis ωD, πB productæ in θ, λ, occurrat. Erit θλ æqualis datæ RS.

Propter rectam RT, rectâ Ao + AO majorem, & rectangula $\overline{RZ \times ZT}$, $\overline{Ao \times AO}$ æqualia, major erit ZT quam AO. Circulus igitur centro A, intervallo ZT, circumferentiæ CO nullibi occurret. Non igitur erit alia præter θλ, datæ RS (majori quam OP) æqualis, circumferentiis OD, PB, ad A vergendo, interjicienda.

Inæquales denique ſint DB, OP, & major eſto OP. (vid. Tab. VI. fig. 2.) Inveniatur minima AΩ, & ducatur Aυ, ut prius. Factiſque PO : Oϑ = BA : AL = BD : CΣ, propter OP majorem quam DB, Oϑ major eſt quam

quam CΣ. Rectangula vero $\overline{OA \times A\Theta}$, $\overline{CA \times A\Sigma}$ æqualia. Major igitur AO quam AΣ. (per Lemma 6. Cor. 2. Segmentorum enim AO, AΘ majus eſt AO, & ſegmentorum CA, AΣ, eſt AΣ majus.) Eâdem vero AΣ minor erit Aσ. Circulus igitur centro A, radio AΣ deſcriptus circumferentiæ σO occurret. Occurſus eſto κ. Juncta Aκ circumferentiis Oτ, Pυ, producta occurrat in Δβ. Ita æquales erunt DB, ΔB, ad contrarias partes minimæ. Facto ut prius SR : RT = BA : AL, factoque $\overline{RZ \times ZT} = \overline{CA \times AV}$, ſi RS non major fuerit quam DB (neque minor quam τυ, idenim ſemper intelligi debet) circulus centro A intervallo RZ deſcriptus circumferentiæ σC occurret, atque circulus alius, centro A, intervallo ZT deſcriptus circumferentiæ σκ occurret. Occurſus ſunto G, γ. Junctæ AG, Aγ, circumferentiis τD, υB, τΔ, υβ, productæ in E, F, φ, ε occurrant. Rectæ EF, εφ ad utraſque partes minimæ, datæ RS ſingulæ æquales erunt.

Quod ſi RS major fuerit quam DB (neque tamen major quam OP) circulus centro A intervallo ZT deſcriptus circumferentiæ κO occurret, & occurſu ſuo poſitionem rectæ unicam definiet, quæ problemati ſatisfaciet; circulus autem centro A radio RZ deſcriptus circumferentiæ CO nullibi occurſurus eſt; quæ quidem omnia facile intellecturus eſt, qui ſuperiora probe percalluerit. Datorum vero ſemicirculorum poſitiones quænam eſſe debent, quæ ſemidiametrorum magnitudines ut rectæ OP, DB, vel æquales ſint, vel ut inæqualium exceſſus penes hanc vel illam ſit, explorare nobis reſtat. Indagini expediendæ opus erit problemate e Pappo.

PRO-

PROBLEMA DIORISTICUM 1^m. (e Pappo.)
(Vid. Tab. VI. fig. 1.)

Semicirculo positione dato AEB, datoque puncto D semicirculum per D describere, qualis est COD, ut ductâ contingente AOP, æquales sint OP, BD.

Factum puta. Esto H centrum circuli COD. Et jungantur HO, BP. Propter æquales BD, PO, erit BD : OA = PO : OA. Et propter angulos ad P, O, rectos, erit PO : OA = BH : HA. Erit igitur BD : OA = BH : HA & $BD^2 : OA^2 = BH^2 : HA^2$, sive $BD^2 : \overline{CA \times AD} : BH^2 : HA^2$. Si fiat igitur BΓ = BD, erit $BD^2 : \overline{CA \times AD} = \overline{\Gamma H \times HD} : CH^2$. Sed CH = HD. Quare $BD^2 : \overline{CA \times AD} = \overline{\Gamma H \times HD} : HD^2$ = ΓH : HD. Bifariam secetur AD in Λ. Et cum AΛ dimidia sit rectæ AD, & CH dimidia rectæ CD, erit HΛ rectæ AC dimidia. Ac proinde $\overline{2AD \times H\Lambda} = \overline{AD \times CA}$. Esto $\overline{2AD \times \Xi} = BD^2$. Propter analogiam supra proditam, (scilicet $\Gamma H : HD = BD^2 : \overline{CA \times AD}$,) erit $\Gamma H : HD = \overline{2AD \times \Xi} : \overline{2AD \times H\Lambda} = \Xi : H\Lambda$. Ac proinde $\overline{\Gamma H \times H\Lambda} = \overline{HD \times \Xi}$. Sed propter BD = BΓ, datamque positione rectam AΓ, datumque punctum B, punctum Γ datum est. Et propter datam magnitudine AΛ, datæ AD dimidiam, datamque eandem positione, datumque punctum A, datum erit & punctum Λ. Datum vero & D. (per hyp.) Et propter datum BD^2, æquale ejus $\overline{2AD \times \Xi}$ datum erit. Data vero 2AD. Ergo & Ξ magnitudine data. Eo igitur res redit, ut datis in rectâ positione datâ, punctis tribus, Λ, D, Γ, quorum Γ, Λ, extrema sunt, datâque magnitudine rectâ Ξ, punctum aliud

(H) inter

(H) inter D, Λ, inveniatur, quod faciat $\overline{\Gamma H \times H\Lambda} = \overline{DH \times \Xi}$.

Factum puta, & adjiciatur utrinque $\overline{\Lambda H \times \Xi}$. Erit igitur $\overline{\Gamma H + \Xi \times \Lambda H}$ ſive $\overline{\Gamma\Lambda + \Xi \times \Lambda H} - \Lambda H^2 = \overline{D\Lambda \times \Xi}$. Datum vero $\overline{D\Lambda \times \Xi}$, propter rectas $D\Lambda$, Ξ magnitudine utraſque datas. Datum igitur $\overline{\Gamma\Lambda + \Xi \times \Lambda H} - \Lambda H^2$. Dataque $\overline{\Gamma\Lambda + \Xi}$. Data igitur & ΛH magnitudine; ſed et poſitione, datumque punctum Λ. Datum igitur & H. Datum vero D. (per hyp.) HD igitur magnitudine data, & ſemicirculus $C\odot D$, cujus centrum H, ſemidiameter HD, poſitione datus erit. Q. E. I.

Vides vero ad compoſitionem requiri, ut ad rectam datam $\overline{\Gamma\Lambda + \Xi}$, datum rectangulum $\overline{D\Lambda \times \Xi}$ applicetur, deficiens quadrato.

Propter $D\Lambda = \frac{1}{2}AD$, & $\overline{2AD \times \Xi} = BD^2$, erit $\overline{D\Lambda \times \Xi} = \frac{1}{4}BD^2$. BD vero ſemper minor quam $\Gamma\Lambda$; minor igitur quam $\overline{\Gamma\Lambda + \Xi}$. Ac proinde $\frac{1}{4}BD^2$, ſive $\overline{D\Lambda \times \Xi}$ ſemper minus erit quam $\frac{1}{4}\overline{\Gamma\Lambda + \Xi}^2$. Problema igitur ſemper componi poterit.

Compositio.

Fiat $B\Gamma = BD$. Biſecetur AD in Λ. Rectis 2AD, BD, tertiam proportionalem cape Ξ. Ad rectam $\overline{\Gamma\Lambda + \Xi}$ applicetur rectangulum $\overline{D\Lambda \times \Xi}$ deficiens quadrato; (quod ſemper poteſt.) Rectæ $\overline{\Gamma\Lambda + \Xi}$ ſegmento minori, æqualis eſto ΛH, (fieri enim non poſſe ut ΛH majori ſegmento æqualis ſit, mox oſtendam.) Centro H intervallo HD circulus ſcribatur COD. Ducatur AO circulum

COD

COD contingens; producta AO circumferentiæ AEB in P occurrat. Æquales erunt OP, BD.

Quod ſi ponatur OP major quam BD, analyſis ut prius inſtituta prodibit $\overline{\Gamma\Lambda+\Xi}\times\Lambda H-\Lambda H^2$ majus quam $\overline{D\Lambda\times\Xi}$. Et ſi OP ponatur minor quam BD, invenietur $\overline{\Gamma\Lambda+\Xi}\times\Lambda H-\Lambda H^2$ minus quam $\overline{D\Lambda\times\Xi}$. Datâ autem $\overline{\Gamma\Lambda+\Xi}$, ſi duarum ΛH, & $\overline{\Gamma\Lambda+\Xi}-\Lambda H$, ΛH ſemper minor ſit, creſcente ſpatio $\overline{\Gamma\Lambda+\Xi}\times\Lambda H-\Lambda H^2$, creſcet neceſſario recta ΛH. (per. Lemma 7.) Dico autem duarum ΛH, $\overline{\Gamma\Lambda+\Xi}-\Lambda H$ minorem eſſe ſemper ΛH. ΛH enim oſtenſa eſt ſemiſſis rectæ AC. AC vero minor eſt quam BD. (per hyp.) Certe igitur ΛH minor quam BD; multo igitur minor quam ΓD, quæ & ipſa minor eſt quam ΓH. Et ipſa ΓH minor quam $\overline{\Gamma H+\Xi}$, ſive $\overline{\Gamma\Lambda+\Xi}-\Lambda H$. Duarum igitur ΛH, $\overline{\Gamma\Lambda+\Xi}-\Lambda H$, minor ſemper eſt ΛH, & proinde creſcente ſpatio $\overline{\Gamma\Lambda+\Xi}\times\Lambda H-\Lambda H^2$, creſcet recta ΛH.

Semicirculi igitur omnes per D tranſeuntes qui centra punctis H, D interjecta habent, rectas OP inter contactus O, & circumferentiam AEB interjectas, majores abſcindunt quam DB; aſt qui centra habent punctis H, A, interjecta, minores abſcindunt. Unicus qui centrum habet H, abſcindit OP, DB æquales.

LIMITES. Caſus 2^us.

(Vid. Tab. 4. fig. 4.) Ducatur AOP ut in caſu priori. AE major eſt quam AP, & AF minor quam AO. FE igitur major quam OP, AE autem minor eſt quam AB, & AF major quam AC. FE igitur minor quam CB,

Om-

Omnium igitur FE maxima eſt CB, OP minima.

Compoſitio eadem ac in caſu priori.

Porro, in caſu utroque, neceſſe eſt ut ſemicirculi ad eaſdem partes ſint baſeos.

INCLINATIO III.

Ad angulum ſemicirculi minoris, a majoris circumferentiâ remotiorem.

Caſus ſunt bini, prout circumferentiæ rectam datam amplexæ, exterior concava, interior convexa, vel concavæ utræque fuerint.

Iiſdem datis ac in inclinationibus noviſſimis, circumferentiis CFD, AEB, interjectam puta EF datæ magnitudinis, quæ ad D vergat. (Vid. Tab. IV. fig. 7, 8.) Occurrat ED circumferentiæ AEB iterum in G. Rectangulum $\overline{ED \times DG}$ datum eſt, dato ſcilicet $\overline{AD \times DB}$ æquale. Ductâ vero ab N rectâ NK ad rectam EG perpendiculari, æquales erunt EK, KG. Factâ igitur EM = DG, æquales erunt MK, KD; et ſi ducatur ab M recta ML ad angulos cum EG rectos, propter parallelas ML, KN, æquales erunt LN, ND. Æquales autem AN, NB. Æquales igitur AL, DB. Data igitur eſt AL magnitudine. (propter DB magnitudine datam.) Data igitur & CL, datarum AC, AL differentia. Data igitur proportio CD, ad CL. Junctâ autem CF, angulus CFD rectus eſt. (propter ſemicirculum CFD) Parallelæ igitur ſunt CF, ML. Ac proinde FM : FD = CL : CD. Data igitur eſt ratio FM ad FD. Duæ igitur rectæ ED, DG, datum rectangulum complexæ ſunt. Et, in fig. 7ª, altera DG, ſive

EM, datâ EF imminuta, ad alteram, ED, eâdem EF imminutam, datam rationem habet.

In fig. autem 8ª. Propter datam rationem FM ad FD, ratio FM ad MD data eſt. Datæ igitur EF & rectæ DG, ſeu æqualis EM, differentia, FM, ad duarum ED, DG differentiam MD, rationem datam habet. Rectæ igitur ED, DG magnitudine utræque datæ ſunt. (In fig. 7ª. per Lemma 3. Caſ. 2. & in fig. 8. per Lemma 5. Caſ. 4. Tab. I. fig. 11.) Datum vero punctum D, & circumferentiæ AEB poſitione data. Rectæ igitur DE, DG poſitione datæ. (per Lemma 2.) Q. E. D.

LIMITES.

In fig. 7. Ducatur per D recta DO, quæ circumferentiam CFD in D contingat. Rectæ DO & circumferentiæ AEB occurſus eſto O. Jungantur HE, HO, quarum HE circumferentiæ CFD in Q occurrat. Propter circulum CFD, $\overline{EF \times ED} + HQ^2 = HE^2$. Et propter angulum HDO rectum, $DO^2 + DH^2 = HO^2$. Sed HO major quam HE, & HO^2 majus quam HE^2. Spatium igitur $DO^2 + DH^2$ ſpatio $\overline{EF \times ED} + HQ^2$ majus. Ablatis igitur æqualibus DH^2, HQ^2, reſtabit DO^2 majus quam $EF \times ED$. Major itaque eſt DO quam ut inæqualium EF, ED, quarum EF minor eſt, media proportionalis ſit; ac proinde minori illâ EF multo major.

Rurſum HE major eſt quam HA. Et HQ = HC. Quare EQ major quam AC, & EF (a fortiori) major.

In fig. 8. Ducatur DO ut prius. Propter angulum ad K rectum major eſt ND quam NK. DO igitur minor quam EK. Multo minor igitur quam EF.

Rur-

Rurſum (propter parallelas CF, NK) CN : CD = FK : FD. Sed CD major eſt quam FD. CN igitur major quam FK, & NB major quam KE. Tota igitur CB, totâ FE major.

In caſu igitur priori omnium EF, minima eſt AC, DO maxima.

In ſecundo, omnium EF maxima eſt, CB, DO minima.

Porro, in caſu priori, neceſſe eſt ut ſemicirculi ad partes baſeos ſint eaſdem, in ſecundo, ad contrarias.

COMPOSITIO.

Eſto RS recta datæ magnitudinis. Fiat AL = DB. Et, in fig. 7, in datâ ratione CL ad CD fiat DB ad DV, & RS ad RT. Si CL major ſit quam CD (ut in fig. 7. N. 1.) erit RT minor quam RS. In hoc caſu ad rectam TS applicetur rectangulum rectangulo $\overline{AD \times DV}$ æquale, ut quadrato excedat. Exceſſûs latitudo eſto SQ. Centro D, intervallo TQ circulus deſcriptus, (ſi rectæ RS longitudo intra limites præfinitos conſiſtat) circumferentiæ AEB occurret. Occurſus eſto E. Juncta DE problemati ſatisfaciet.

Aſt ſi CL minor ſit quam CD (ut in fig. 7. N. 2.) erit RT major quam RS. Et, in hoc caſu, ad rectam ST applicetur rectangulum $\overline{AD \times DV}$ excedens quadrato. Exceſſûs autem latitudo eſto TQ. Centro D intervallo TQ circulus deſcriptus (modo rectæ RS longitudo intra limites præfinitos ſe contineat) circumferentiæ AEB occurrat. Occurſus eſto E. Juncta DE problemati ſatisfacit. Hæc facile intelliget, qui analyſin præmiſſam cum Lemmate 3°. Caſ. 2. attentius contulerit.

In fig. 8. In datâ ratione CL ad LD, fiat DB ad BV, & RS ad RT. Ad datam RT applicetur rectangulum AD × DV, ut excedat quadrato. Exceſſûs latitudo eſto TQ. Centro D intervallo TQ circulus deſcriptus (ſi rectæ RS longitudo intra limites præfinitos conſiſtat), circumferentiæ AEB occurſurus eſt. Occurſus eſto E. Juncta DE & producta problemati ſatisfacit.

Caſus autem uterque ad Lemma 5^m^ revocari poſſet. Sed caſûs prioris compoſitio per Lemma 3^m^. longe commodius conſit.

INCLINATIO IV.

Ad Angulum ſemicirculi minoris, circumferentiæ exteriori propiorem.

Caſus ut præcedentis bini.

Iiſdem vero ac in præcedente datis, circumferentiis AEB, CFD interjectam puta EF datæ magnitudinis, quæ ad C vergat. (Vid. Tab. IV. fig. 9, 10.) Recta autem EC circumferentiæ AEB, intra baſim productæ, occurrat iterum in G. Rectangulum EC × CG datum, dato ſcilicet AC × CB æquale. Ductâ autem ut prius ad EG perpendiculari NK, factâque EM = GC, æquales erunt MK, KC. Ductâ igitur ML ad angulos cum EG rectos, æquales erunt CN, NL. Æquales igitur AC, BL. Data igitur BL. Data igitur & CL, & datarum CD, CL ratio data. Junctâ vero DF, propter DF, ML parallelas, CF : CM = CD : CL. Data igitur ratio CF ad CM. Rectæ igitur duæ EC, CG datum rectangulum complexæ ſunt, alterius autem, EC, & minoris datæ, EF, in fig. 9, dif-

differentiæ, CF, ad duarum EC, CG (ſeu æqualis EM) differentiam CM, ratio data. Rectæ igitur EC, CG magnitudine utræque datæ ſunt (per Lemma 5. Caſ. 4. N. 2.)

In fig. 10ª. Propter rationem CF ad CM datam, convertendo ratio CM ad MF data eſt. Eſt autem MF rectæ CG, ſeu æqualis EM, & minoris datæ EF differentia; CM vero rectarum EC, CG differentia eſt. Datum vero $\overline{EC \times CG}$ rectangulum. Rectæ igitur EC, CG magnitudine utræque datæ ſunt (per Lemma 5. Caſ. 4. N. 2.)

Datæ igitur, in utrâque figurâ, rectæ CG, CE magnitudine. Datum vero punctum C, & circumferentia ABG poſitione data. Rectæ igitur CG, CE poſitione datæ. (per Lemma 2.) Q. E. I.

COMPOSITIO et LIMITES Caſûs prioris. (fig. 9.)

Ducatur per C recta co quæ circumferentiam CFD in c contingat, circumferentiæ autem AEB in O occurrat, eidemque intra baſim continuatæ iterum in P. Eſto RS recta datæ magnitudinis quæ ad C vergendo circumferentiis CD, OB interponi debet. Fiat BL = AC. In datâ ratione CD ad CL fiat BC ad BV, necnon RS ad RT. Propter CL ſemper majorem quam CD, major ſemper erit BV quam BC, & RT quam RS. Ad rectam RT applicetur rectangulum rectangulo $\overline{AC \times CV}$ æquale, ut deficiat quadrato. Defectûs latitudo eſto RZ vel ZT. Circulus centro C intervallo alterutro RZ vel ZT deſcriptus circumferentiæ AP ſi occurrat, punctum G unum atque alterum ſignabit. (ut ex analyſi præmiſſâ & Lemmate 5. Caſ. 4. N. 2. ſatis patet.)

Ad

Ad compoſitionem vero requiritur, ut ad datam RT datum rectangulum $\overline{AC \times CV}$ applicetur deficiens quadrato.

Id ut fieri poſſit, neceſſe eſt ut $\frac{1}{4}$ RT^2 minus non ſit quam $\overline{AC \times CV}$. Neque igitur RT^2 minus quam $\overline{4AC \times CV}$. Propter analogiam $CD : CL = RS : RT$, $RT^2 = \frac{CL^2}{CD^2} RS^2$. Quare $\frac{CL^2}{CD^2} RS^2$ minus eſſe non debet quam $\overline{4AC \times CV}$. Neque igitur RS^2 minus quam $\frac{CD^2}{CL^2} \overline{4.AC \times CV}$. Fiat igitur $CD : CL = CL : CQ$. Fiat CY rectæ AC quadrupla. Fiat $CQ : CD = CY : CX$. Diametro XV ſemicirculus conſtitutus, rectæ CP in Ω occurrat. Erit CΩ omnium RS minima[a]. Fiat $CD : CL = C\Omega : C\Phi$. Erit CΦ omnium RT minima. Biſectâ denique CΦ in Ψ, centro C intervallo CΨ circulus deſcriptus circumferentiæ AP ſi occurrat, occurſu ſuo poſitionem minimæ definiet.

Videamus igitur, an circulus ille circumferentiæ AP ſemper occurſurus ſit: ſive (quod eodem redit) an recta AΦ vel major eſſe poterit quam CP, vel minor quam CA.

Sint, æquales CΨ, CP, (vid. Tab. VII. fig. 1.) Æqualia igitur $C\Psi^2$, CP^2. Sed $C\Psi^2 = \overline{AC \times CV}$, (ut ex analyſi manifeſtum eſt) & $CP^2 = \overline{AC \times CB}$ (propter circulum.) Æqualia igitur rectangula $\overline{AC \times CV}$, $\overline{AC \times CB}$. Æquales igitur CV, CB &, $BV = 2BC$. Et propter analogiam $BC : BV = CD : CL$, $CL = 2CD$. Quare $CD = DL$. Sed $AC = LB$. Quare $AD = DB$. Rectam igitur AB, punctum D biſecat. Aſt N biſecat. Congruunt igitur D, N. Id

[a] Quæ vel hoc modo brevius invenietur. Duabus AC, CV cape mediam proportionalem quæ dicatur M. Ut CL ad CD fiat M ad quartam, quæ dicatur N. Erit 2N minima omnium RS.

in

eſt circulus interior per exterioris centrum trajicit. Et in hoc caſu erit co æqualis cΩ, & minima omnium quæ ad c vergendo circumferentiis cD, OB interponi poterunt; eritque DB earundem maxima.

Ponantur autem inæquales cΨ, cP, & eſto cΨ major. (Vid. Tab. VII. fig. 2.) Rectangulum AC × CV = cΨ². (ut ex analyſi & Lemmatis 5ⁱ. unde pendet caſu, manifeſtum eſt). Sed propter cΨ majorem quam cP, erit cΨ² majus quam cP². Majus igitur AC × CV quam PC², ſive AC × CB. CV igitur major quam CB. Ac proinde BV major quam 2BC, & propter analogiam CD : CL = BC : BV, CL major quam 2CD. DL igitur major quam DC, & additis BL, CA æqualibus, BD major quam DA. Punctum igitur D, punctis N, A interjacet. Et in hoc caſu co major, quam cΩ, minima tamen omnium quæ ad c vergendo circumferentiis, CFD, OB interjici poterunt; DB vero earundem maxima.

Sunto autem inæquales cΨ, cP, & cP major eſto. In hoc caſu oſtendetur DL minor quam DC, eodem modo ac in priori oſtenſa eſt DL major. Minor igitur BD quam DA, & punctum D punctis B, N interjacet (Vid. Tab. VII. fig. 3.)

Videndum eſt an cΨ, minor quamcP, minor eſſe poterit quam AC, & quando. Ponantur autem æquales. (Vid. Tab. VII. fig. 4.) Erunt igitur cΨ², AC² æqualia. Sed cΨ² = AC × CV. Quare AC² = AC × CV. Puncta igitur C, V congruunt. Erit igitur BC : BA = CD : CL. Quare invertendo, & convertendo, BA : AC = CL : LD. Et dividendo, BC : CA = CD : DL. Unde dato puncto c, facile invenietur circulus, qui BA, BV, necnon AC, cΨ æquales præſtat. Scilicet ſi data CL ſecetur in D in datâ ratione BC ad CA, & diametro CD ſemicirculus conſtituatur.

Im

In hoc caſu omnium quæ circumferentiis CD, OB, ad c vergendo, interponi poterunt, minima erit DB, (quæ & cΩ æqualis erit), earundem vero co maxima.

Inæquales vero ponantur AC, cΨ, & cΨ minor eſto. (Vid. Tab. VII. fig. 5.) Minus igitur $c\Psi^2$ quam AC^2. Minus igitur $\overline{AC \times CV}$ quam AC^2, & punctum V punctis, C, A interjacet. Habet igitur BC ad minorem quam BA rationem eam quam CD ad CL. Ratio igitur BC ad BA minor quam ratio CD ad CL. Invertendo ratio BA ad BC major quam CL ad CD. Convertendo ratio BA ad AC minor quam CL ad LD. Denique dividendo, ratio BC ad CA minor quam CD ad DL.

In hoc caſu DB major erit quam cΩ, minima tamen omnium quæ, ad c vergendo, circumferentiis CD, OB interponi poterunt. Et co earundem maxima

Ponantur denique inæquales AC, cΨ. Et cΨ major eſto quam Aϵ, cum minor tamen ſit quam CP. (Vid. Tab. VII. fig. 3. 6, 7.)

Propter cΨ majorem quam AC, majus eſt $c\Psi^2$, ſive $\overline{AC \times CV}$, quam AC^2. Puncti igitur V, in CA ultra A productâ, locus erit. In hoc caſu, habet BC ad majorem quam BA rationem eam quam CD ad CL. Ratio igitur BC ad BA major quam ratio CD ad CL. Invertendo, convertendo, & dividendo, ratio BC ad CA major eſt quam CD ad DL. (Minor vero erit CV quam CB, propter cΨ minorem quam CP. Idenim ſupra oſtenſum.) Et in hoc caſu, circulus centro c intervallo cΨ deſcriptus circumferentiæ AP occurret. Occurſus eſto σ. Juncta σc & producta circumferentiis CD, OB in τ, υ occurrat, erit τυ = cΩ, & omnium minima quæ ad c vergendo circumferentiis

cumferentiis CD, OB interponi poterunt. Omnium vero quæ circumferentiis $C\tau$, $O\upsilon$, maxima OC, quæ circumferentiis τD, υB maxima omnium DB.

Rectis autem OC, DB vel æqualibus esse licet, vel inæqualibus, et inæqualium huic vel alteri esse majori.

Compositio Repetita.

(Tab. VII. fig. 1.) Transeat primo circumferentia interior per centrum exterioris. In hoc casu, recta data quæ, ad C vergendo, circumferentiis CD, OB interponenda est, neque major esse debet quam DB, neque minor quam CO: confit autem problema modo peculiari. Namque (in Tab. VII. fig. 1.) propter angulum DFG rectum, punctumque D centrum circuli GAE, æquales sunt GF, FE. Datâ igitur FE, dabitur ejus dupla GE; et problema componetur ope 2^i. problematis libri præcedentis, rectam datæ magnitudinis GE, in circulo dato, inscribendo, ut ad punctum datum C vergat.

Circumferentia vero interior per exterioris centrum non transeat. Punctum igitur D vel punctis A, N, vel B, N interjacebit. Interjaceat primo punctis A, N. (vid. Tab. VII. fig. 2.) Esto RS recta datæ magnitudinis, quæ, ad C vergendo, circumferentiis CD, OB interponi debet. Rectæ RS neque minori esse licet quam CO, neque majori quam DB. Fiat $BL = AC$. Fiat ut CD ad CL ita RS ad RT, & BC ad BV. Ad rectam RT, applicetur rectangulum $= \overline{AC \times CV}$ deficiens quadrato. Defectûs latitudo esto RZ, vel ZT, quarum RZ minor esto. (Atque in sequentibus idem intelli-

gatur). Circulus centro C intervallo RZ deſcriptus circumferentiæ AP occurret (oſtendetur enim RZ minor quam CP, & major quam AC, ſimili ratiocinio, ac in Tab. V. fig. 1. oſtenſa eſt RZ major quam AC, & minor quam AO.) Occurſus eſto G. Juncta GC & producta problemati ſatisfaciet. Recta autem ZT major erit quam CP. Circulus igitur centro C intervallo ZT deſcriptus circumferentiæ AP nullibi occurſuruseſt. Rectæ igitur, quæ problemati ſatisfaciet, poſitio in hoc caſu unica.

Punctum vero D, punctis N, B interjectum eſto. (Tab. VII. fig. 3, 4, 5, 6, 7.) Fiat, ut prius, $BL = AC$. In datâ ratione BC ad CA, ſecetur CL in k. Si puncta k, D congruant, ut in Tab. VII. fig. 4, rectæ RS neque majori eſſe licet quam CO, neque minori quam DB, per ſuperius oſtenſa. Facto autem $RS : RT = CD : CL$, factoque etiam $\overline{RZ \times RT} = CA^2$, circulus centro C intervallo ZT deſcriptus circumferentiæ AP occurſurus eſt. Occurſus eſto G. Juncta GC & producta problemati ſatisfaciet.

Circulus vero centro C intervallo RZ deſcriptus circumferentiæ AP nullibi occurſurus eſt. Poſitio igitur rectæ problemati ſatisfacturæ unica.

Punctum vero k cadat inter puncta C, D. (Tab. VII. fig. 5.) Ratio igitur BC ad CA, minor eſt quam CD ad DL. Et in hoc caſu rectæ RS neque minori eſſe licet quam DB neque majori quam CO. Facto autem BC ad BV, necnon RS ad RT, ut CD ad CL, factoque etiam $\overline{RZ \times ZT} = \overline{AC \times CV}$, circulus centro C intervallo ZT deſcriptus circumferentiæ AP occurret. Occurſus eſto G. Juncta CG & producta problemati ſatisfaciet. Circulus vero centro C radio RZ de-

deſcriptus circumferentiæ AP non occurret. Rectæ igitur quæ problemati ſatisfacit poſitio & in hoc caſu unica.

Cadat denique punctum k inter D, L. In hoc caſu, ratio BC ad CA major eſt quam CD ad DL. Inveniantur CΩ, CΦ, & ducatur σCυ, ita ut τυ, CΩ æquales ſint, ac proinde τυ minima. Quæ, quâ ratione efficienda ſint, ex præmiſſis ſatis patet. Rectæ OC, DB vel æquales erunt, vel non. Sunto primo æquales. (Tab. VII. fig. 6.) Rectæ igitur RS neque minori eſſe licet quam τυ, neque majori quam CO, ſive DB. Facto autem BC ad BV, necnon RS ad RT, ut CD ad CL, factoque etiam $\overline{RZ \times ZT} = \overline{AC \times CV}$, circulus centro C intervallo RZ deſcriptus circumferentiæ Aσ occurret; circulus antem centro C intervallo ZT deſcriptus, circumferentiæ σP occurret. Occurſus ſunto G, γ. Juncta CG, Cγ problemati utræque ſatisfacient.

Sunto autem inæquales OC, DB. (Tab. VII. fig. 3.) Si DB major ſit quam CO, (fig. 3.) fiat CD : CL = CO : PΘ. Circulus centro C intervallo CΘ deſcriptus circumferentiæ Aσ occurret. Occurſus eſto π. Juncta πC & producta circumferentiis CD, OB occurret. Occurſus ſunto κ, ω. Æquales erunt Co, κω.

Si CO major ſit quam DB, (Tab. VII. fig. 7.) fiat CD : CL = DB : AΣ. Circulus centro C, intervallo CΣ deſcriptus circumferentiæ σP occurret. Occurſus eſto α. Juncta αC & producta circumferentiis CD, OB occurrat. Occurſus ſunto ∫, β. Æquales erunt DB, ∫β. In fig. autem 3. & 7, rectæ RS neque minori eſſe licet quam τυ, neque majori quam duarum

DB, CO major. Si duarum DB, CO minore minor ſit, circuli centro C, intervallis RZ, ZT deſcripti circumferentiæ AP utrique occurrunt, ad partes vero minimæ contrarias; et, occurſu gemino, geminam poſitionem rectæ problemati ſatisfacientis definient.

Quod ſi recta RS duarum DB, CO minorem ſuperârit, circulus in figura 3ª. centro C, intervallo RZ deſcriptus circumferentiæ $A\pi$ occurſurus eſt. Et, occurſu ſuo, poſitionem rectæ unicam, quæ problemati ſatisfaciet, definiet. In fig. autem 7ª. Circulus centro C intervallo ZT deſcriptus circumferentiæ $P\alpha$ occurret, & poſitionem ut prius unicam occurſu definiet.

Hæcce autem omnia, eodem prorſus modo, ex analyſi & Lemmatis 5ⁱ. unde pendent caſu derivantur, ac Limites varii atque compoſitiones caſus primi, Inclinationis penultimæ: atque horum demonſtrationes labore leviſſimo vel Tyro quiſque pro ſe conficiet, ſi priora illa probe intellexerit. Fuſius igitur exponere abſtineo, ne peritioribus tædium afferam. Reſtat autem, ut breviter oſtendam, qui debent eſſe centrorum ſitus, quæve diametrorum magnitudines, ut rectæ OC, DB vel æquales ſint, vel ut inæqualium exceſſus penes hanc vel illam ſit.

Problema Dioristicum 2ᵐ. (e Pappo.)

(Vid. Tab. VII. fig. 6.) Semicirculo AEB & baſi AB, poſitione datis, datoque puncto D, inter B & centrum dati ſemicirculi N, ſemicirculum qualis eſt CFD

CFD conſtituere, ut ductâ contingente CO, æquales ſint CO, DB.

Dividatur BA in C, ut ſit rectangulum $\overline{BC \times CA} = DB^2$. (Quod ſemper poteſt cum DB minor ſit quam NB.) Diametro CB ſemicirculus conſtitutus problemati ſatisfacit.

Semicirculi vero omnes per D deſcripti ſemicirculo CFD (in fig. 6ª.) minores, faciunt rectas CO majores quam DB; contra ſemicirculi majores quam eſt CFD in fig. 6ª, per D tranſeuntes, faciunt rectas CO minores quam DB. Quæ quidem omnia luce clariora.

Compositio. Caſus 2ⁱ.

Eſto RS recta datæ magnitudinis, cujus æqualis ad C vergendo, circumferentiis AEB, CFD interponi debet. (Vid. Tab. IV. fig. 10.) Fiat BL = AC. Fiat ut LD ad LC, ita RS ad RT, & BC ad BV. Ad rectam RT applicetur rectangulum rectangulo $\overline{AC \times CV}$ æquale, ut quadrato deficiat. Defectûs latitudo eſto RZ. Ducatur per C, OCP ut in caſu præcedente. Circulus centro C intervallo alterutro RZ vel ZT deſcriptus circumferentiæ AO ſi occurat, punctum E unum atque alterum ſignabit.

Limites.

RS^2 minus eſſe non debet quam $\frac{LD^2}{LC^2} 4\overline{AC \times CV}$. Fiat igitur LD : LC = LC : LQ. Cape CY rectæ AC quadruplam. Fiat LQ : LD = CY : CX. Semicirculus

micirculus diametro VX conſtitutus rectæ CP in Ω occurrat. Erit CΩ minima omnium, quæ per C tranſeuntes, circumferentiis AEB, CFD unquam interponi poterunt[a]. Fiat LD : LC=CΩ : CΦ. Si CΦ in Ψ bifariam ſecetur, circulus centro C intervallo CΨ deſcriptus, circumferentiæ AO ſi occurrat, occurſu ſuo poſitionem minimæ definiet.

Occurret vero circulus iſte circumferentiæ AO, ſi recta CΨ neque minor fuerit quam AC, neque major quam CO.

Si puncta A, V congruant, æquales erunt CA, CΨ. Si punctum V punctis C, A interjectum ſit, minor erit CΨ quam CA. In hiſce caſibus, erit AD omnium EF minima, & OC earundem maxima.

Quod ſi neque congruant puncta A, V, neque V punctum punctis A, C, interjaceat, erit V in CA ultra A productâ. Recta enim BV rectâ BC minor eſſe nequit. (per conſtruct.)

Eſto V in CA ultra A productâ. Si CV, CB æquales ſint, (quod eveniet ſi puncta D, N congruant) æquales erunt CO, CΨ. Si CV major ſit quam CB, (quod eveniet ſi punctum D, punctis L, N interjaceat) CΨ major erit quam CO. Et in hiſce caſibus, recta CO omnium EF minima erit, AD maxima.

At ſi CV minor ſit quam CB (quod eveniet ſi punctum D, punctis N, C interjaceat) CΨ minor erit quam CO. Eadem vero CΨ (cum V ſit in CA ultra A productâ) major quam CA. Circulus igitur centro C intervallo CΨ deſcriptus circumferentiæ AO, inter A, O puncta occurret, & occurſu ſuo poſitionem minimæ definiet, rectæ vero CO, AD, ad ſuas quæque

[a] Quæ vel brevius invenietur modo ſequente. Rectis AC, CV cape mediam proportionalem, quæ dicatur M. Fiat ut LC ad LD, ita M ad N. Erit 2N minima omnium quæ per C, &c.

partes

partes minimæ, maximæ erunt, et gemina erit problematis conſtructio ad utraſque partes minimæ, niſi forſan, exiſtentibus OC, AD inæqualibus, data RS, maximâ inæqualium OC, AD, neceſſario minor, minore tamen major fuerit. Ita enim unica futura eſt problematis conſtructio.

Porro puncto D inter A, N, dato, ſemicirculus qui CO, AD æquales præſtat, invenietur ut in caſu priori, datam AB ita in C ſecando, ut rectangulum $\overline{AC \times CB}$ dato AD^2 æquale ſit. Quod ſemper poteſt, cum AD minor ſit quam ſemiſſis rectæ AB.

Hæcce, indagine ut in priori caſu inſtitutâ, quivis colliget. Porro neceſſe eſt ut ſemicirculi in priori caſu ad eaſdem ſint partes baſeos, in ſecundo ad contrarias.

Status Semicirculorum Quartus.

Circulis nullibi occurrentibus, neutro incluſo. (Vid. Tab. VIII. fig. 1, 2, 3, 4.)

Semicirculi duo AEB, DFC poſitione dati ſint, in baſi eâdem DCAB. Circuli autem nullibi occurrant, & neuter alterum includat.

Definitio.

Hiſce poſitis Angulos ſemicirculorum ut D, B, baſeos dico extremos. Angulos ut C, A, baſeos medios.

Inclinatio I.

Ad Angulum alterutrum baſeos extremum.

Caſus ſunt bini, prout circumferentiæ datam rectam amplexæ, vel convexæ utræque, vel altera convexa altera concava fuerit. (Vid. Tab. VIII. fig. 1, 2.)

Circumferentiis AEB, CFD, interjectam puta EF, datæ magnitudinis, quæ ad B vergat. Recta BF circumferentiæ

cumferentiæ DFC occurrat iterum in G. Datum eſt rectangulum $\overline{GB \times BF}$ dato $\overline{DB \times CB}$ æquale. Eſto H centrum circuli DFC, ductâque HK ad rectam FG perpendiculari, æquales erunt GK, KF. Factâ igitur GM=EF, æquales erunt MK, KE, & ſi a puncto M ducatur ML ad angulos cum FM rectos, æquales erunt LH, HA. (Junctâ namque AE, parallelæ erunt AE, ML, propter angulum ad E rectum.) Sed æquales DH, HC. Æquales igitur LD, CA. Data igitur LD magnitudine, totaque proinde LB. Et datarum LB, BA, ratio data eſt. Sed LB : BA=MB : BE. Data igitur ratio MB ad EB. Duæ igitur rectæ GB, BF datum rectangulum amplexæ ſunt. Et alterius BG, datâ FE, ſeu æquali GM, auctæ, ad alteram BF, eâdem datâ imminutam, ratio data eſt. Rectæ igitur BG, BF, magnitudine utræque datæ ſunt. (per Lemma 3. Caſ. 3.) Data igitur BF magnitudine. Ergo & poſitione (per Lemma 2.) Q. E. D.

Limites.

In figura 1ª. ducatur a B, recta BO quæ circumferentiam DFC contingat. Contactus eſto O. Recta autem BO circumferentiæ AEB in P occurrat.

BC minor eſt quam BF, & BA major quam BE. CA igitur minor quam FE.

Rurſum BF minor eſt quam BO, & BE major quam BP. EF igitur minor quam OP.

In figura 2ª. ducatur BPO ut prius. Jungantur HO, AP. Propter angulos, ad O, & P, K & E rectos, OP : OB=HA : HB=KE : KB. Sed KB, major quam OB. KE igitur major quam OP. Unde FE multo major quam OP.

Rurſum

Lib II.
Tab. IV.

Fig. 1.

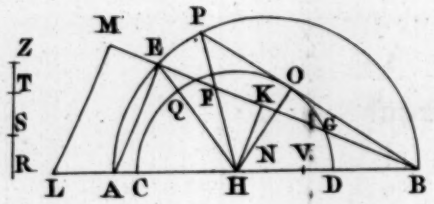


Fig. 2.

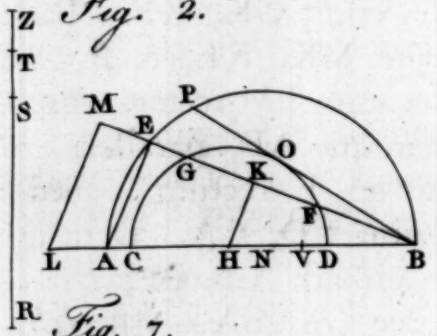


Fig. 3.

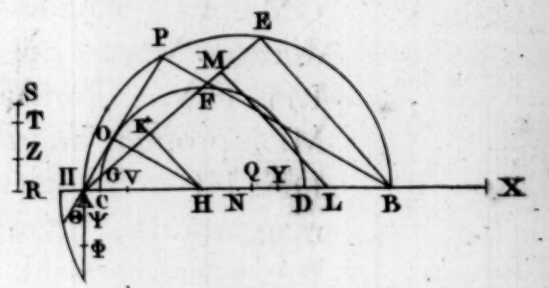


Fig. 4.

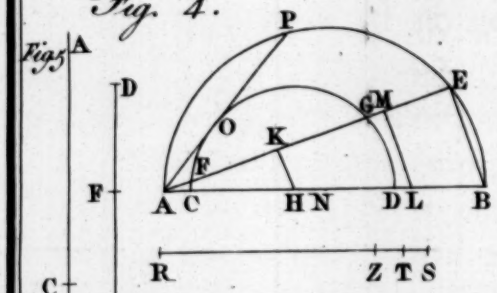


Fig. 7. N°. 2.

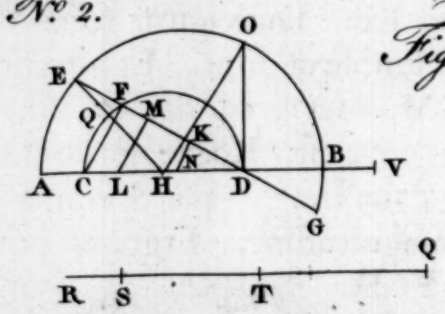


Fig. 8.

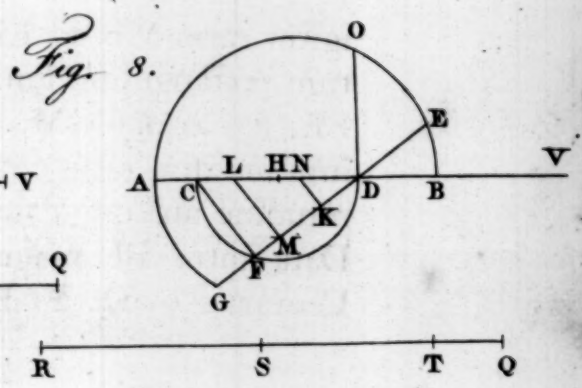


Fig. 7. N°. 1.

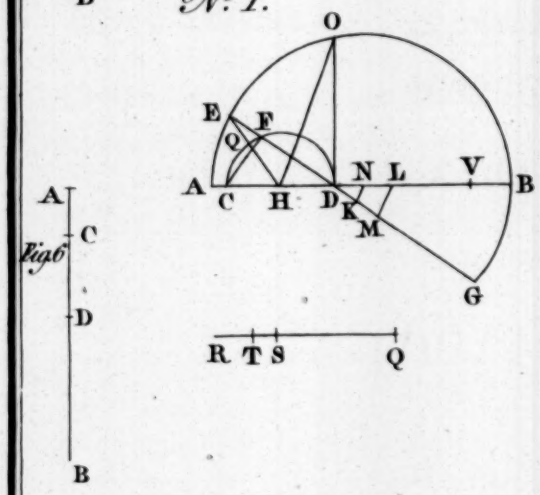
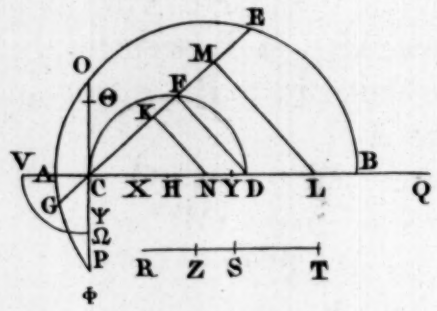


Fig. 9.



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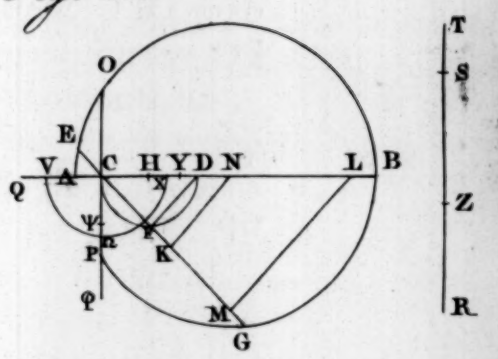


Lib. II.
Tab. V.

Fig. 3.

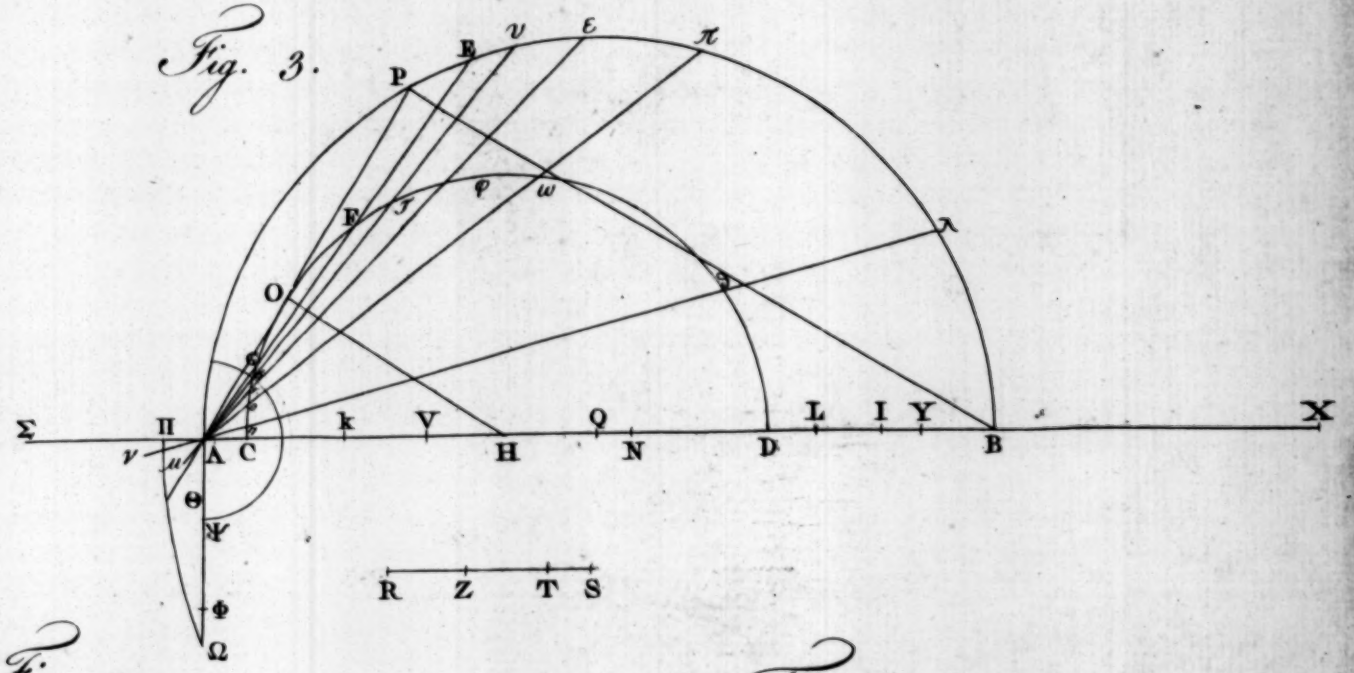


Fig. 2.

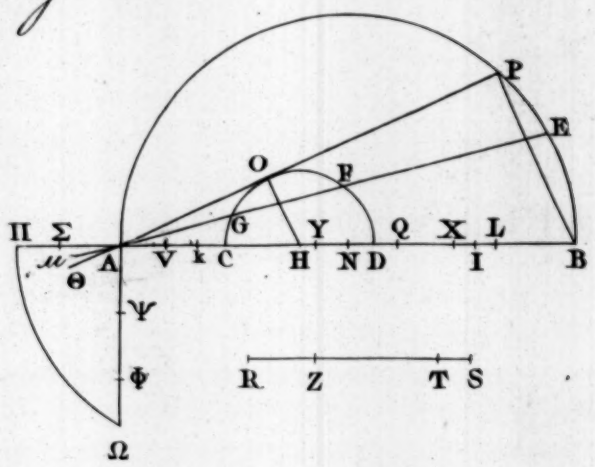
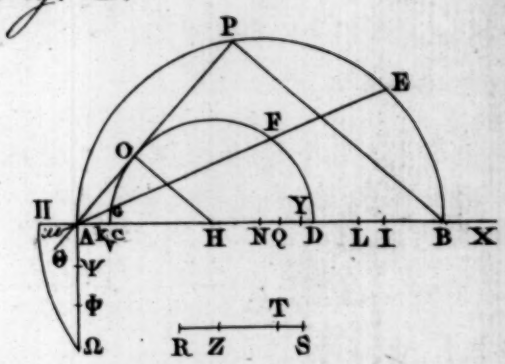


Fig. 1.



Lib. II.
Tab. VI.

Fig. 2.

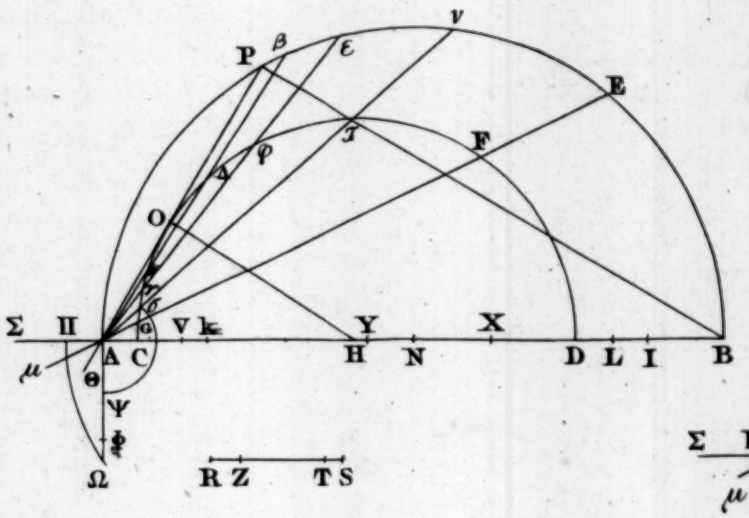
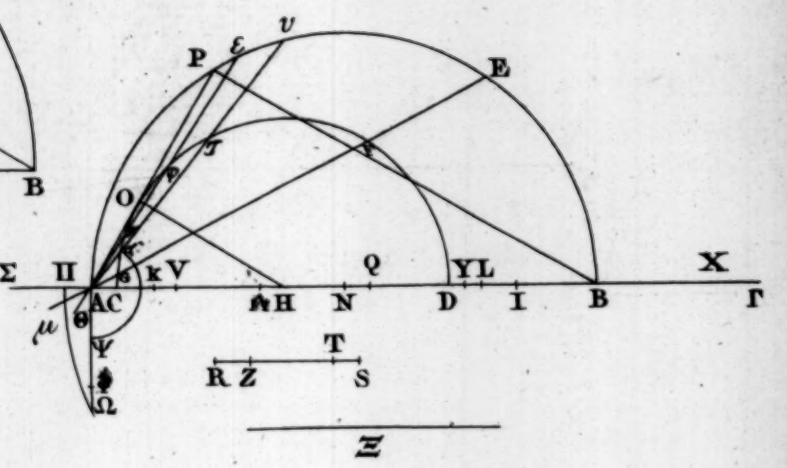


Fig. 1.



Lib. II.
Tab. VII.

Fig. 1.

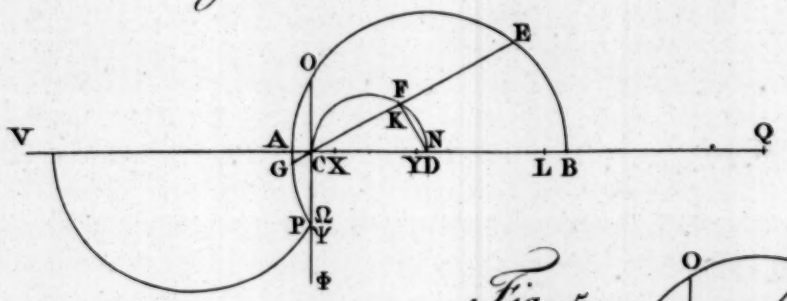


Fig. 6.

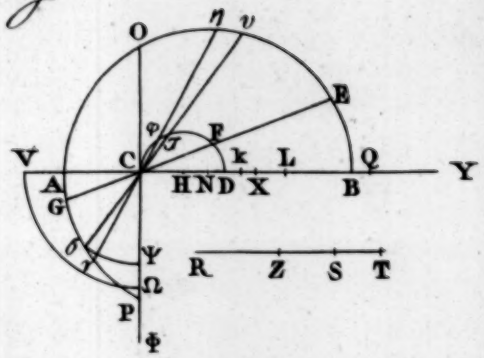


Fig. 5.

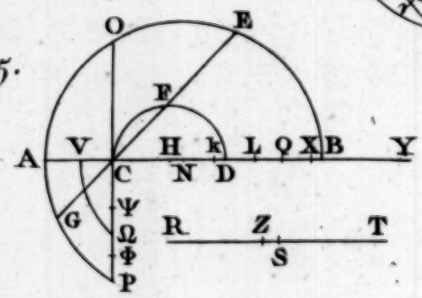


Fig. 7.

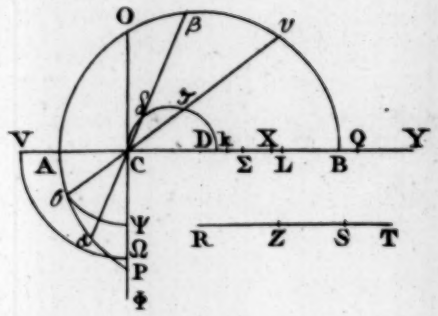


Fig. 2.

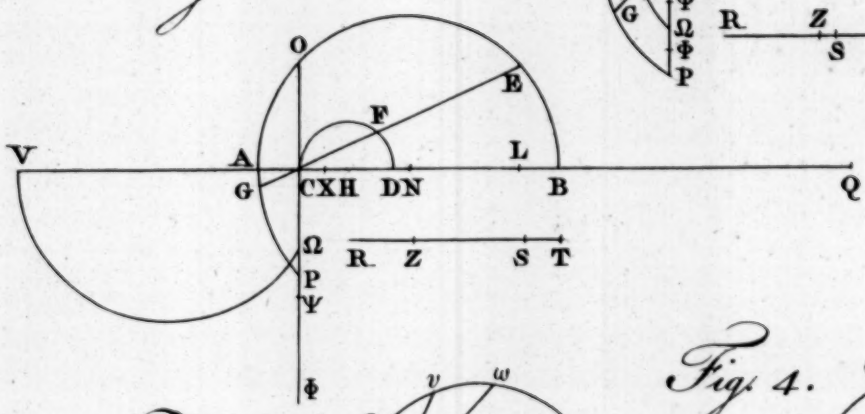
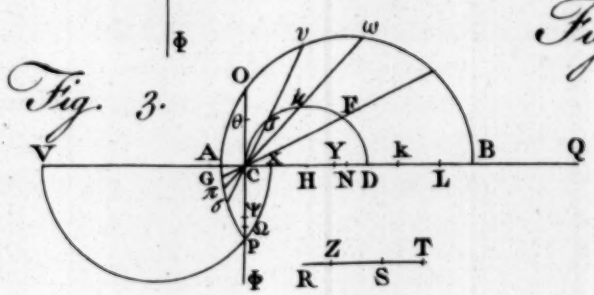
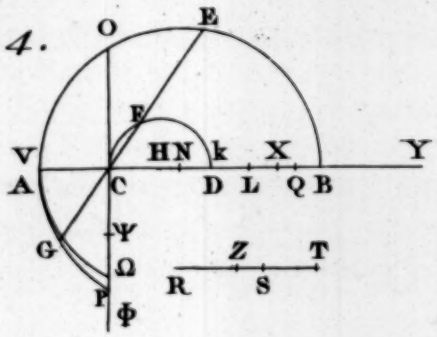


Fig. 3.



Figs. 4.



Rurſum HB major quam KB. HA igitur major quam KE. Sed & HD major quam KF. Tota igitur DA, totâ FE major.

In caſu igitur priori, omnium FE minima eſt CA, OP maxima.

In caſu ſecundo, omnium FE minima eſt OP, DA maxima. Porro in caſu utroque, neceſſe eſt ut ſemicirculi ad partes baſeos ſint eaſdem.

Compositio.

Detur RS, in caſu utroque, intra limites præſtitutos. Factâ DL=CA, fiat ut LB ad BA ita BC ad BV, necnon RS, ad RT. Ad rectam ST applicetur rectangulum $\overline{DB \times BV}$ ut quadrato excedat. Exceſſûs latitudo eſto TZ. Centro B intervallo SZ circulus deſcriptus circumferentiæ DGC occurret. Occurſus eſto F. Juncta BF, & producta problemati ſatisfacit.

Inclinatio II.

Ad angulum alterutrum Baſeos medium.

Caſus bini, prout circumferentiæ, datam rectam amplexæ, altera convexa, altera concava, vel concavæ utræque fuerint. Iiſdem ac prius poſitis, circumferentiis AEB, CFD interjectam puta EF, datæ magnitudinis, quæ ad A vergat. (Vid. Tab. VIII. fig. 3, 4.) Recta vero AF circumferentiæ CFD, in G iterum occurrat. Datum igitur rectangulum $\overline{AG \times AF}$ dato $\overline{DA \times AC}$ æquale. Ductâ autem ab H, centro circuli DFC, ad rectam GF perpendiculari HK, æquales erunt FK, KG. Factâque GM=AF, æquales erunt MK, KA; ductâque ML ad angulos cum MF rectos,

æquales erunt LH, HA. Sed DH=HC. Quare LD =CA. Data igitur eſt LD, totaque proinde LA. Et datarum LA, AB ratio data. Sed LA : AB=MA : AE. (Junctâ namque BE, parallelæ ſunt ML, BE, propter angulos ad M, E rectos.) Rectæ igitur duæ AG, AF datum rectangulum amplexæ ſunt. Alterius autem AF, & majoris datæ FE, differentiæ AE, ad duarum ſummam AM, ratio data eſt. Rectæ igitur utræque AF, AG magnitudine datæ ſunt. (per Lemma 5. Caſ. 3. No. 2.) Quare & poſitione (per Lemma 2.)

Compositio et limites casus prioris.

(Tab. VIII. fig. 3.)

Recta datæ magnitudinis eſto RS. Ducatur ab A recta quæ circumferentiam CFD contingat. Contactus eſto O. Rectæ autem AO & circumferentiæ AEB occurſus eſto P. Fiat DL=AC. Fiat ut BA ad AL ita AC ad CV, necnon RS ad RT. Ad rectam RT applicetur rectangulum $\overline{DA \times AV}$ deficiens quadrato. Defectûs latitudo eſto RZ vel ZT. Circuli centro A intervallis RZ, ZT deſcripti circumferentiæ OD, ſi occurrant, punctum G unum atque alterum definient.

Ad Compoſitionem igitur requiritur, ut ad datam rectam RT, rectangulum DA × AV applicetur, deficiens quadrato. Id ut fieri poſſit, neceſſe eſt $\frac{1}{4}$ RT² minus non ſit quam $\overline{DA \times AV}$, ſive RT² non minus quam $\overline{4\,DA \times AV}$. Sed propter analogiam RS : RT= AB : AL, $RT^2 = \frac{AL^2}{AB^2} RS^2$. Ac proinde $\frac{AL^2}{AB^2} RS^2$ non debet minus eſſe quam 4 DA × AV. Neque igitur RS² minus

minus quam $\frac{AB^2}{AL^2}$ 4 DA × AV. Fiat igitur AL : AB = AB : AQ. Fiat LA : AQ = 4 DA : AX. Diametro VX ſemicirculus deſcriptus, rectæ per A circumferentiam AEB contingenti, in Ω, occurrat. Erit AΩ minima omnium, quæ, ad A vergendo, circumferentiis AEB, CFD unquam interponi poterunt. [a] Fiat vero AΩ : AΦ = AB : AL. Atque AΦ erit omnium RT minima. Biſecetur AΦ in Ψ. Circulus centro A intervallo AΨ deſcriptus, circumferentiæ DO ſi occurrat, occurſu ſuo, poſitionem minimæ definiet.

Circulus autem centro A intervallo AΨ deſcriptus circumferentiæ DO ut occurrat, neceſſe eſt AΨ neque minor ſit quam AO, neque major quam AD.

Propter AV majorem quam AC, rectangulum $\overline{DA \times AV}$ rectangulo $\overline{DA \times AC}$ ſemper majus eſt. Sed $A\Psi^2 = \overline{DA \times AV}$, & $AO^2 = \overline{DA \times AC}$. Majus igitur $A\Psi^2$ quam AO^2, & recta AΨ rectâ AO ſemper, major. Fieri igitur nequit ut AO, AΨ æquales ſint vel ut inæqualium ſit AΨ minor.

Ponantur autem æquales AΨ, AD. Rectangulum igitur $\overline{DA \times AV}$ $(= A\Psi^2) = AD^2$. Congruunt igitur puncta D, V. Ac proinde BA : AL = AC : CD. Datis igitur ſemicirculo CFD, & puncto A, ſemicirculus qui æquales facit CD, CV, necnon AD, AΨ facile inveniendus eſt. Scilicet ſi fiat DC : CA = $\overline{DA + AC}$: AB, & diametro AB ſemicirculus deſcribatur.

Si talis eſſet ſemicirculus AEB in ſchemate tertio, Tab. VIII. æquales eſſent CB, AΩ, & CB omnium

[a] Quæ vel hoc modo brevius invenietur. Duabus DA, AV cape mediam proportionalem, quæ dicatur M. Fiat ut AL ad BA ita M ad quartam, quæ dicatur N. Erit 2 N minima omnium quæ ad A &c.

minima eſſet, quæ circumferentiis CΩ, AEB, ad A vergendo, interponi poſſent, recta autem OP earundem maxima.

Ponantur vero inæquales AΨ, AD, & AΨ major. Majus igitur $\overline{AD \times AV}$ quam AD^2. AV igitur major quam AD. Et BA ad AL minorem rationem habebit quam AC ad CD. Sive ratio DC ad CA minor eâ quam habet $\overline{AD+AC}$ ad AB. Quare, in hoc caſu, AB minor erit ſemidiametro circuli qui AD, AΨ æquales præſtat. CB vero, in hoc caſu, major erit quam AΩ, minima tamen omnium quæ circumferentiis CO, AEB, ad A vergendo, interponi poterunt, et OP earundem maxima.

Denique ſi inæqualium AC, AΨ, minor ſit AΨ, colligetur AB ſemidiametro circuli qui AD, AΨ æquales præſtat major, ſimili prorſus ratiocinio, ac, in caſu noviſſimo, oſtenſa eſt minor. Aſt, in hoc caſu, circulus centro A intervallo AΨ deſcriptus circumferentiæ DO occurſurus eſt. Occurſus eſto σ. Juncta σA & producta circumferentiis CO, AEB in punctis τ, υ occurrat. Æquales erunt τυ, & AΩ, & τυ minima omnium quæ ad A vergendo circumferentiis CO, AEB interponi poterunt. Quæ vero circumferentiis TO, υP maxima omnium OP, quæ circumferentiis Cτ, υB maxima omnium CB.

Recta igitur RS, rectâ τυ non debet eſſe minor, neque duarum CB, OP majore major. Si duarum CB, OP minore RS minor ſit, binæ erunt rectæ quæ problemati ſatisfacient, ad partes minimæ diverſas. Quod ſi RS duarum CB, OP minorem ſuperarit, unica recta problemati ſatisfaciet, inter majorem & minori æqualem. Porro quoties rectas OP, CB æquales eſſe contigerit, gemina erit problematis conſtructio.

Hæcce

Hæcce omnia quivis intelliget, qui caſus 1^i, Inclinationis ſecundæ, Status tertii, limites & conſtructiones probe percalluerit.

Quinam vero centrorum H, N ſitus eſſe debent, quæve diametrorum magnitudines, ut rectæ OP, CB æquales ſint, vel ut inæqualium exceſſus penes hanc vel illam ſit, reſtat ut exponam.

PROBLEMA DIORISTICUM 3^m.

Semicirculo DFC poſitione dato, datoque puncto A, in baſi DC productâ, ſemicirculum qualem AEB deſcribere, ut ſi recta OA, circumferentiam datam CFD in O contingens, producatur, donec circumferentiæ AEB occurrat, (ut in P) OP, CB æquales ſint. (Tab. VIII. fig. 3.)

Si æquales ſint CB, OP, erit PO : OA = BC : OA. Junctis vero BP, HO, propter angulos ad O & P rectos, PO : OA = BH : HA. Ac proinde BH : HA = BC : OA & permutando BH : BC = HA : AO. Data vero ratio datarum HA, AO. Data igitur ratio BH ad BC. Data igitur & HC ad CB. Data vero HC magnitudine. Data igitur & BC magnitudine. Sed & poſitione, punctumque C datum. Datum igitur & B, & recta AB magnitudine data. Ac proinde ſemicirculus AEB, magnitudine & poſitione datus.

COMPOSITIO.

Fiat ut HA — AO : AO = HC : CB. Componendo erit HA : AO = HB : BC. Ratio autem HA ad AO ratione HA ad AC minor eſt. Ratio igitur HB ad BC ratione HA ad AC minor. Et dividendo ratio HC ad CB ratione HC ad

CA

CA minor. CB igitur major quam CA. Punctum igitur B (quod fieri debet) in recta CA productâ ſitum eſt. Semicirculus diametro AB deſcriptus problemati ſatisfaciet.

Si CB, OP ponantur inæquales, & CB major, major erit ratio BC ad OA quam PO ad OA, ſive BH ad HA. Permutando, ratio BC ad BH major erit quam OA ad HA. Invertendo & dividendo, ratio HC ad CB minor quam HA — OA ad AO. CB igitur major eſt eâ ad quam HC rationem habet quam HA — OA ad AO. Ac proinde AB major eſt diametro ſemicirculi qui OP, CB æquales præſtat. (per problematis dioriſtici analyſin.) Quod ſi inæqualium, CB, OP, minor fuerit CB, ſimili ratiocinio, colligetur AB minor diametro ſemicirculi qui æquales præſtat.

Eſto igitur AEB ſemicirculus qui OP, CB æquales præſtat, ope problematis hujuſce dioriſtici deſcriptus. Semicirculi omnes per A tranſeuntes ſemicirculo AEB majores, faciunt CB majorem quam OP. Omnes vero eodem AEB minores minorem faciunt CB quam OP. Unicus AEB, rectas CB, OP æquales præſtat.

LIMITES Caſus 2^i. (Tab. VIII. fig. 4.)

Ducatur AOP ut in fig. 3^a. Propter angulos ad K, E rectos, HB major eſt quam KE. Sed & HD major quam FK. Tota igitur DB totâ FE major. Rurſum propter angulus ad O, P, (junctis HO, BP) rectos, KA : KE = HA : HB = OA : OP. Sed KA major quam OA. Quare KE major quam OP. Et (a fortiori,) FE eâdem OP major.

Omnium igitur FE minima eſt OP, DB maxima.

Porro, tam in hoc caſu quam priori, neceſſe eſt ut ſemicirculi ad partes baſeos ſint contrarias.

Hujuſce

Hujuſce vero caſûs compoſitio eadem ac prioris.

Circulorum nullibi occurrentium caſus confecimus univerſos. Univerſi ſunt duodecim. Status nempe duplex, altero incluſo vel neutro; primi Status Inclinationes quatuor, ſecundi duæ; cujuſque vero Inclinationis caſus duo, juxta diviſionem admodum generalem. Nam ſi tot caſus numeraveris quot ſunt limitum varietates, Inclinationis 2ᵃ. Status primi, ſex fient Caſus. Inclinationis autem 4ᵃ. caſus octo; Inclinationis ſecundæ, Status ſecundi, caſus ſex: unde circulorum nullibi occurrentium caſus univerſi colligerentur viginti ſex.

Status Semicirculorum Quintus.

Circulis Secantibus.

Semicirculi duo AEB, CFD (Vid. Tab. VIII. fig. 5, 6, 7, 8. & Tab. IX.) in baſi eâdem DACB, poſitione dati ſint. Circumferentiæ vero DFC, AEB, vel ſe mutuo vel oppoſitas, quæ integras ſcilicet circumferentias complent, ſecanto.

Definitio.

Angulos D, B, ut in ſtatu præcedente, baſeos dico extremos. Angulos autem A, C, baſeos medios.

Inclinatio I.

Ad Angulum alterutrum baſeos extremum.

Ut recta, circumferentiis intercepta, ad angulum alterutrum baſeos extremum vergat, neceſſe eſt ut ſemicir-

culi

culi ad eaſdem ſint partes baſeos ; atque inde fiet, ut ſibi mutuo occurſuri ſint. Occurrant (ſecando ſcilicet) in H. Inclinatio autem datæ rectæ ad B fiat.

Caſus varii erunt, pro vario ſitu puncti A.

Locus Puncti A Primus.

Eſto K centrum circuli DHC. (Vid. Tab. VIII. fig. 5, 6, 7, 8. & Tab. IX. ſig. 1, 2, 3, 4, 5, 6.) Circumferentia BHA, per punctum K trajicit, vel non. Primum trajiciat. (Vid. Tab. VIII. fig. 5, 6.) Congruunt igitur puncta A, K. Junctis igitur BH, HA, rectus eſt angulus BHK, ac proinde recta BH circumferentiam DHC, in H, contingit.

Rectæ igitur datæ, circumferentiis DHC, AHB interjectæ, ſitus duplex erit, prout circumferentiis AH, CH, vel AH, DH, interpoſita fuerit. Caſûs autem uterque unâ conſtructione confit. Circumferentiis enim AH, HC (in fig. 5.) vel AH, HD (in fig. 6.) interjectam, puta EF datæ magnitudinis, quæ ad B vergat, & circumferentiæ DFC iterum in G occurrat. Juncta KE, angulus KEB rectus erit (propter ſemicirculum AEB.) Æquales igitur FE, EG. Data igitur FG, datæ FE dupla. Confit igitur problema ope prop. primæ Libri 1^i, rectam nimirum, datæ duplam, in dato ſemicirculo DHC aptando, quæ ad punctum extra datum, B, vergat.

Data vero FE, ſemidiametro circuli DHC non debet eſſe major.

Locus puncti A secundus.

Circumferentia AHB per K non tranſeat. Juncta igitur BH, circumferentiam CHD non continget. (Vid. Tab.

Tab. VIII. fig. 7, 8. & Tab. IX. fig. 1, 2, 3, 4, 5, 6.) Circumferentiæ igitur CHD in alio puncto, præter H, occurret. Occurrat in Q. Ducatur a B, recta BO, quæ circumferentiam CHD contingat. Contactus esto O. Rectæ autem BO & circumferentiæ AEB occursus esto P. Punctum O, aut D, H punctis, aut C, H interjectum erit. Esto primum inter D, H. (Tab. VIII. fig. 7, 8. & Tab. IX. fig. 1, 2.) Recta igitur BH circumferentiæ HD producta occurret, & punctum A punctis K, B interjectum erit. Et in hoc situ puncti A, situs datæ rectæ, circumferentiis AHB, CHD, interjectæ, quadruplex erit; nimirum inter circumferentias AH, HC, vel HA, HD, vel HP, HO, vel HP, OQ.

Cas. 3, 4, 5, 6. Circumferentiis igitur AH, HC (Tab. VIII. fig. 7.) AH, HD, (Tab. VIII. fig. 8.) PH, HO, (Tab. IX. fig. 1.) PH, OQ, (Tab. IX. fig. 2.) interjectam puta EF, datæ magnitudinis, quæ ad B vergat. Occurrat autem FE circumferentiæ DHC iterum in G. Ductâ KL, a centro K, in FG perpendiculari, æquales erunt FL, LG. Factâ igitur GM = FE, ductâque MN ad angulos cum FG rectos, ratiocinio in prioribus toties usurpato, æquales reperientur DN, AC, dataque proinde DN, ac datarum NB, BA ratio data. Data igitur (quæ eadem est) BM, ad BE ratio. Duæ igitur rectæ BF, BG datum rectangulum amplexæ, magnitudine utræque datæ sunt. (per Lemma 3. cas. 3.) Quare & positione (per Lemma 2.)

LIMITES.

In Tab. VIII. fig. 7, BA major est quam BE, & BC minor quam BF. AC igitur major quam EF.

Omnium igitur EF, in caſu 3°, maxima eſt AC.

In fig. 8, Duc KR ad QH perpendicularem. Junctis AE, AH, propter angulos ad R, H, L, E, rectos, LE : EB =KA : AB= RH : HB. Sed AB major quam EB. KA igitur major quam LE. Sed & KD major quam LF. Tota igitur AD, totâ EF major.

Rurſum propter HB minorem quam EB, RH minor erit quam LE. Sed et RQ minor quam LF. Tota igitur HQ, totâ EF minor.

Omnium igitur EF in caſu 4° maxima eſt AD, HQ minima.

In Tab. IX. fig. 1, Propter BF minorem quam BO, & & BE majorem quam BP, erit FE minor quam PO.

In caſu igitur 5° omnium FE, maxima eſt OP.

In Tab. IX. fig. 2ª, Junctis KO, AP AH, AE, ductâque KR ad HQ perpendiculari, propter analogias RH : HB=KA : AB=LE : EB=OP : PB, rectamque EB minorem quam HB, & majorem quam PB, erit LE minor quam RH, & major quam OP. Sed & FL minor quam RQ.

Tota igitur FE minor quam QH, & major quam OP.

In caſu igitur 6°, omnium FE, minima eſt OP, HQ maxima.

Compoſitiones autem ex analyſi ſponte proveniunt.

Locus Puncti A Tertius.

Eſto punctum O inter C, H. (Vid. Tab. IX. fig. 3, 4, 5, 6.) Recta igitur BH circumferentiæ C, O, inter C, O occurret, & punctum A punctis K, D interjacet.

Situs

Situs igitur rectæ, circumferentiis datorum ſemicirculorum interjectæ, iterum quadruplex erit, prout circumferentiis AH, QC, (fig. 3.) ſive HP, OQ, (fig. 4.) ſive HP, HO, (fig. 5.) ſive denique HA, HD, (fig. 6.) interpoſita fuerit.

Caſ. 7, 8, 9, 10. Conſtructiones & analyſis eædem ac quatuor præcedentium.

De Limitibus videamus.

LIMITES.

In fig. 3ª. Propter BE minorem quam BA, & BF majorem quam BC, EF minor erit quam AC.

Rurſum propter BE majorem quam BH, & BF minorem quam BQ, EF major eſt quam HQ.

Omnium igitur EF, in Caſu 7°, maxima eſt AC, minima HQ.

Similiter, in fig. 4ª, oſtendetur EF minor quam HQ, & major quam OP.

Omnium igitur EF, in caſu 8°, maxima HQ, PO minima.

In figª. 5ª, eſto Y centrum circuli AEB. Recta AY, major erit quam AK, ne circulus AEB circulo DEC includeretur. (quod hypotheſi contrarium eſſet.) Jungantur igitur EK, PK, quarum EK circumferentiæ DFC in h occurrat. Rectangulum $\overline{EF \times EG} + hK^2 = EK^2$. Et $PO^2 + OK^2 = PK^2$. Sed PK major quam EK (per 7. 3 Elem. Quare $PO^2 + OK^2$ majus quam $\overline{EF \times EG} + hK^2$. Et ablatis æqualibus, hK^2, OK^2, majus erit PO^2 quam $\overline{EF \times EG}$. Sed PO minor quam EG, propter BP minorem

rem quam BE, & BO majorem quam BG. Eadem igitur PO major quam EF.

Omnium igitur EF, in caſu 9°, maxima eſt OP.

In fig^a. 6^a, jungantur DF, FC. Angulus DFC rectus eſt. DFG igitur obtuſus. Ducta igitur, a D, recta Dh in BF perpendicularis, rectæ BF occurret ultra F productæ. Major itaque Eh quam EF. Sed hE : EB = DA : AB. Ac proinde hE minor eſt quam DA, propter EB minorem ſcilicet quam AB. Et FE minor eſt quam hE. Multo minor igitur quam DA. Omnium igitur FE, in caſu 10°, maxima eſt AD.

INCLINATIO II.

Ad angulum alterutrum baſeos medium.

Iiſdem poſitis ac prius, inclinatio fiat ad angulum A.

Semicirculi vero ad partes baſeos aut eaſdem erunt aut contrarias.

Sint primo ad eaſdem. (Vid. fig. 7. & 8.) Circumferentiis CD, AB interjectam puta FE, datæ magnitudinis, quæ ad A vergat. Punctum vero A, vel C, K punctis, ut in fig. 7, vel K, D ut in fig. 8, interjaceat. Et occurrat, in caſu utroque, recta EF circulo CFD iterum in G. Rectangulum $\overline{FA \times AG}$ datum eſt, dato $\overline{CA \times AD}$ æquale. Ductâ a K in rectam FG perpendiculari KL, factâque GM = AF, ductâque a puncto M, rectâ MN ad angulos cum FG rectos, æquales erunt DN, AC. Data igitur DN, ac proinde NA, & datarum NA, AB ratio data. Data igitur ratio MA ad AE.

Rectæ

Rectæ igitur AF, AG, magnitudine utræque datæ (per Lemma 5. Caſ. 2.) Quare & poſitione (per Lemma 2.)

LIMITES.

In figura 7, AE minor eſt quam AB, & AF major quam AC. (per 7. 3 Elem.) Ergo FE minor quam CB.

In figura 8ª. Junctis CF, FD erit angulus CFD rectus. Acutus igitur CFA. Ductâ igitur a puncto C, rectâ Ch ad FA perpendiculari, punctum h, punctis F, A interjacebit. Major itaque Eh quam EF. Sed propter angulos ad E, h rectos, Eh : EA = CB : BA. Sed EA minor quam AB. Eh igitur minor quam CB. Ac proinde EF multo minor.

Omnium igitur EF in utroque caſu CB eſt maxima.

Caſ. 2. Sint vero ſemicirculi ad partes baſeos contrarias (vid. fig. 9, 10.) & circumferentiis CD, AB interjectam puta FE, datæ magnitudinis, quæ ad A vergat. Recta autem AF circumferentiæ DFC iterum occurrat, ut in G. Ductâ a K ad FG perpendiculari KL, factâque GM = AF, ductâque MN ad angulos cum FG rectos, ratiocinio toties uſurpato, æquales oſtendentur DN, CA, dataque proinde DN, & NA, & datarum NA, AB ratio data, cui æqualis eſt ratio MA ad AE. Rectæ autem FA, AG rectangulum datum (dato $\overline{DA \times AC}$ æquale) comprehendunt. Magnitudine igitur utræque datæ ſunt. (per Lemma 5. Caſ. 4. Tab. I. fig. 11.) Quare & poſitione (per Lemma 2.)

LIMITES.

Qualiſcunque ſitus ſit puncti A, ſive inter K, C, ut in

in fig. 9. ſive inter K,D, ut in fig. 10ª, ducatur per A recta AO circumferentiam AEB in A contingens. Rectæ vero AO, & circumferentiæ DFC occurſuus ſit O; propter angulum ad L rectum, KA major eſt quam KL. AO igitur minor quam LG. Multo minor igitur quam LE, quæ & ipſa minor eſt quam FE.

Rurſum propter angulos ad L, E rectos, KB major eſt quam LE. Et DK major quam FL. Tota igitur DB totâ FE major.

Omnium igitur FE, in utrâque figurâ, maxima eſt DB, AO minima.

Secantium caſus univerſi ſunt duodecem. Inclinationes nempe duæ. Primæ Inclinationis caſus decem, ſecundæ duo.

Problematis vero caſus univerſi ſunt triginta ſex, juxtad iviſionem maxime generalem. Concentricorum caſus quatuor, (ſepoſitis ſcilicet ultimis tribus problematis primi inclinationibus, utpote quæ problemati Apolloniano neutiquam videntur incluſæ.) Contingentium octo. Nullibi occurrentium duodecem. Secantium duodecem. Quod ſi nullibi occurrentium caſus (accuratius diſtinguendo) numeraveris viginti ſex, univerſi erunt quinquaginta.

Scholion.

Hæcce fuit, quantum ex Pappi Lemmatibus, conjecturâ aſſequi poſſum, ipſius Apollonii, nobiliſſimi problematis, per caſus omnes, ſolutio. Pappi namque Lemmata, præter quod hujuſce libri primum eſt, & e problematibus dioriſticis, quæ nobis tria ſunt, priora duo, hiſce inquam exceptis, quæ in ſecundum Inclinationum librum Pappus lemmata ornavit, eo omnia ſpectant, ut, in problematum compoſitionibus, rectas DE, FM, (Tab. III. fig. 2, 3. 7, 8.) BF, EM (Tab. III. fig. 4, 5.) FM, BG (Tab. IV. fig. 1, 2) FM, AG (Tab. IV. fig. 3, 4.) EM, DG (Tab. IV. fig. 7, 8.) EM, CG (Tab. IV. fig. 9, 10.) GM, FE, (Tab. VIII. fig. 1, 2.) GM, FA (Tab. VIII. fig. 3, 4.) GM, FE (Tab. VIII. fig. 7, 8. & Tab. IX. fig. 1, 2, 3, 4, 5, 6, 7, 8.) FM, AG (Tab. IX. fig. 9, 10.) æquales evincant. Alias longe, & admodum perplexas, caſûum variorum conſtructiones, ineunte ſeculo decimo ſeptimo, in *Apollonio* ſuo *Redivivo*, Ghetaldus edidit; quas calculo algebraico derivatas (ut ex opere ejus poſtumo, *De Reſolutione & Compoſitione Mathematicâ*, ſat liquido conſtat) in Apollonio ſuo, ſynthetice demonſtrare aggreſſus eſt, diſſimulato quâ usus ſit analyſi; credo, quod uberrimam ejus explicationem in tempus aliud diſtulit, & in alio illo opere tradendam animo decrevit; quod ne vivo prodiret, Viri præclari immatura mors prohibuit. Quam vero in conſulto faciunt, qui, in rebus geometricis, veterum analyſi poſthabitâ, ad Algebram ſemper confugiunt, is demum ſentiet, qui Ghetaldi conſtructiones cum hiſce noſtris contulerit.

tulerit. Figuræ ſcilicet, quotquot ſunt, lineis conſtant vel pluribus, vel curvâ fortaſſe unâ. Linearum vero plurium, ut & partium ejuſdem curvæ variarum, relatio duplex eſt: *Quantitate* etenim diſtinguuntur, & *Situ*. Meras quantitatis relationes, *meras* autem dico, quæ ex ſitu nullo modo pendent, optimo certe compendio Algebra expendit, & ex notis ignotas mirâ facilitate promit.—Situs linearum varios dignoſcere, & cum alias omnes, tum & ipſius quantitatis relationes, ſi quæ ex ſitu oriundæ, vel lineis ipſis, vel figuris quas lineæ claudunt, intercedant, explorare, id, ni fallor, Geometriæ munus eſt. In problematibus pleriſque, Solutio ex utrâque relatione pendet; Situs dico & Magnitudinis: vel compendioſius ſaltem ex utrâque junctim, quam ex hac vel illâ ſeorſim elicienda eſt. Poſitiones vero linearum diverſas, interſectiones, contactus, angulos, flexuras, & quæ plurima inde forte conſequantur, quæ vix fieri poteſt ut prudentiorem paullo *Geometram* vel lateant vel fugiant, *calculus* ſæpiſſime præterire ſolet. Unde haud raro evenit, ut qui æquationibus concinnandis nimium ſe delectari ſinunt, exitu operis, quod brevi ſane, & nullo fere negotio, conficiunt, in conſtructiones aut nullas plane incidunt, aut perplexas adeo & laborioſas, ut omni prorſus utilitate careant. Algebræ autem ita demum legitimus erit, in Geometriâ, uſus, ſi in *rebus calculi ſolis* adhibeatur. Neque tamen Geometris auctor eſſem, ut eo ſtudium atque artem potiſſimum intendant, ut ad calculum rem quamque propoſitam deducant. Imo contrarium ſuadeo. Conſtructiones enim ſimpliciſſimas & facillimas eſſe, quæ ex poſitionibus partium figuræ petitæ ſunt, multiplex me experientia

Lib. II.
Tab. VIII.

Fig. 1.

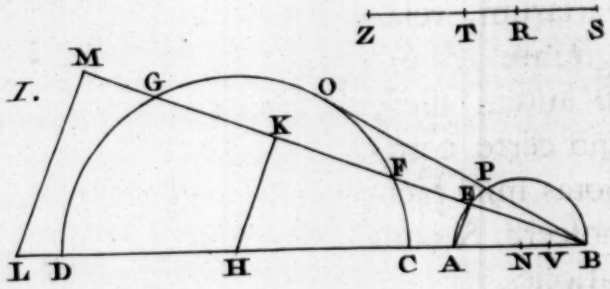


Fig. 2.

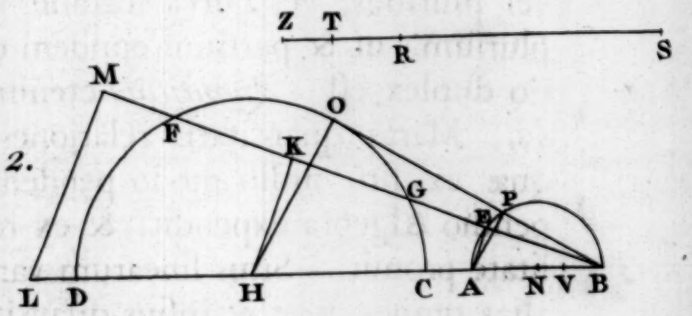


Fig. 3.

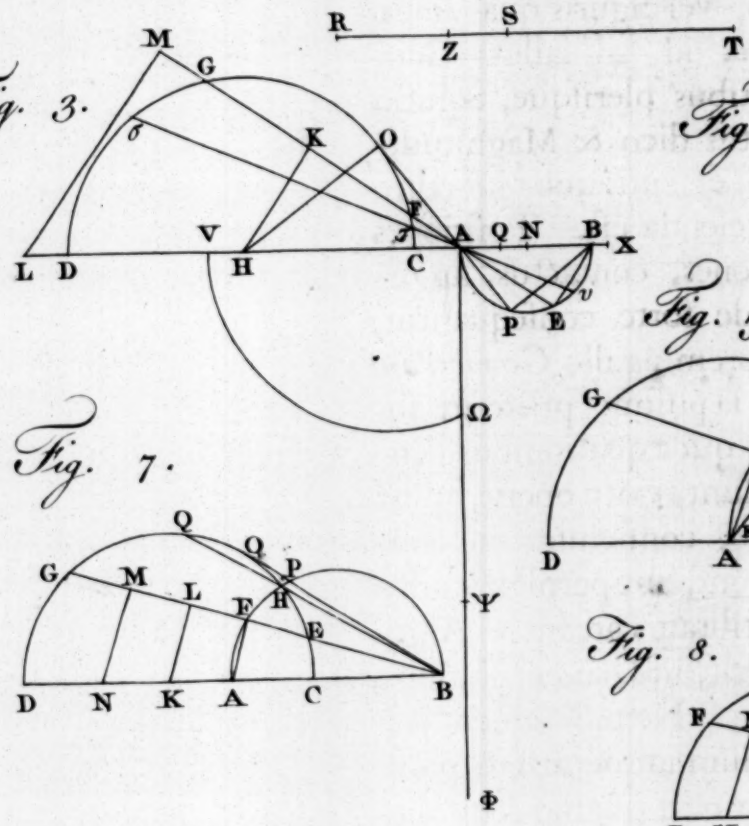


Fig. 4.

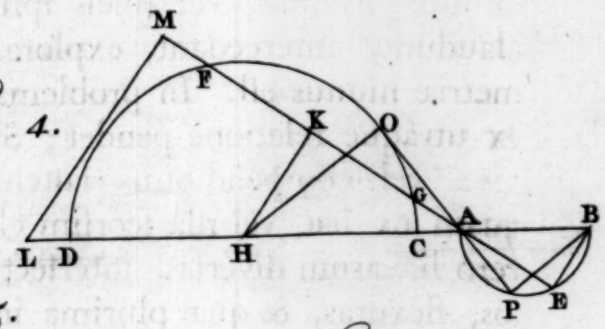


Fig. 5.

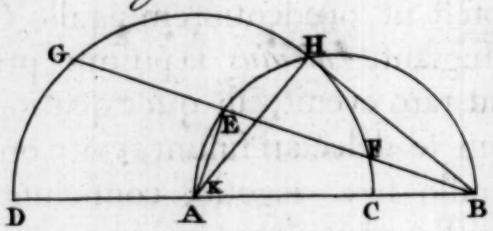


Fig. 6.

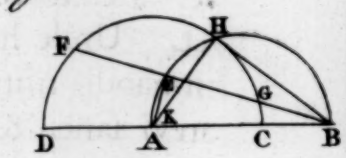
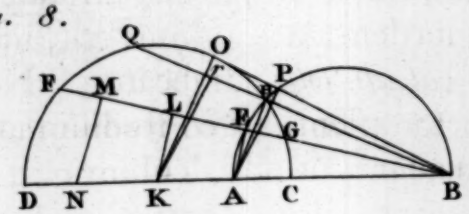


Fig. 7.

Fig. 8.



perientia docuit. Sed cum, indagine rite inſtitutâ, omnibus datorum & quæſitorum, tam ſitûs quam magnitudinis, relationibus diligenter perpenſis, eo tandem, ſuâ quaſi ſponte, res devenerit, ut mero calculo ulterius proſequenda ſit, nollem ſane, cæcâ adeo veneratione, antiqua amplecti, ut, in tali negotio, recentiora Algebræ compendia ingratus ſpernerem. Neque minus imperite & inepte faciunt, meo equidem judicio, qui in rebus calculi Algebrâ uti nolunt, quam qui in rebus Graphicis, Geometriâ, id eſt Graphicês ſcientiâ, multum valere juſſâ, maxima quæque problemata mero calculo aggreſſi, Geometrarum munere præclare ſe defunctos exiſtimant, dummodo æquationes utcunque concinnaverint. Subjungere viſum eſt pauca quædam problemata, quorum analyſi, quæ de diverſis Geometriæ & Algebræ muneribus differui, dilucidiora fient.

PROBLEMA I.

In rectas tres poſitione datas, nec parallelas nec in uno aliquo puncto occurrentes, a dato puncto rectam ducere, cujus ſegmenta, datis intercepta, datam mutuo rationem habeant.

(Vid. Tab. IX. fig. 11.) Datæ ſint rectæ tres triangulum ABC claudentes; et a dato puncto D, ductam puta DEGF, cujus ſegmenta EG, GF datam rationem habeant.

Per E agatur EH parallela rectæ BC. Erit HC : CF = EG : GF. Data igitur ratio HC ad CF. Data vero ejuſdem HC ad EB ratio, datæ ſcilicet AC ad AB, (propter parallelas EH, BC) æqualis. Ratio igitur EB, ad CF data. (per 8 dat.) Rectis igitur AB, AF poſiti-

one datis, recta DF, per datum punctum D ducta, ſegmenta abſcindit BE, CF, punctis datis B, C terminata, in datâ ratione. Recta igitur DF poſitione data eſt, per Problema Generale Apollonii de Ratione abſcindendâ.

Problema II.

Rectis duabus, non parallelis, poſitione datis, per datum punctum rectam ducere, quæ, cum datis, triangulum claudat dato ſpatio æquale. (Tab. 9. fig. 12.)

Datæ poſitione ſint rectæ AB, AC. A dato puncto D ductam puta DEF, quæ triangulum, EAF, cum datis, dato ſpatio æquale claudat. Propter rectas poſitione datas, AB, AC, datus eſt angulus EAF. Rectanguli igitur $\overline{EA \times AF}$ ad triangulum EAF ratio data (per 66. dat.) Triangulum vero EAF magnitudine datum (per hyp.) Datum igitur $EA \times AF$. Datumque punctum D (per hyp.) Res igitur redit ad Problema Generale Apollonii de Spatio abſcindendo.

Simili negotio, nec majori, alia ejuſdem generis problemata reſolutionem capiunt. Ut ſi recta per datum punctum ducenda eſſet, quæ aream trianguli, magnitudine & poſitione dati, in datâ ratione divideret: vel cum tribus rectis poſitione datis, neque omnibus parallelis, datum parallelogrammum vel trapezium clauderet: atque id genus alia, in quibus plebeii æquationum artifices immane quantum deſudârunt.

Problema III.

Datis in triangulo rectilineo (ABC) angulo verticali (C)

(C) ſummâ laterum (AC+CB), ſummâ perpendicularis & baſeos (AB+CD,) triangulum deſcribere. (Tab. IX. fig. 13.

Propter angulum ACB datum, data eſt ratio ſpatii $\overline{AC+CB}^2 - AB^2$ ad Triangulum. (per 67 dat.) Triangulum vero ſpatii $\overline{AB \times CD}$ ſubduplum eſt. Data igitur eſt ratio $\overline{AC+CB}^2 - AB^2$ ad $\overline{AB \times CD}$. Ponatur $\overline{AC+CB} = EF$, & $\overline{AB+CD} = EG$ & $AB = EH$. Eo igitur res redit, ut rectis EF, EG magnitudine datis, recta EG ita ſecetur in H, ut $\overline{EH \times HG}$ ad $EF^2 - EH^2$ rationem datam habeat. Quod calculi ſcilicet negotium eſt, cum partes ejuſdem rectæ magnitudine ſolâ diſtinguantur [a]. Pono igitur EF ſeu $\overline{AC+CB} = a$. EG ſeu $\overline{AB+CD} = b$. EH ſeu $AB = x$. HG ſeu $CD = b - x$. Et datam $EF^2 - EH^2$ ad $\overline{EH \times HG}$ rationem, R ad S. Erit igitur (pro $EF^2 - EH^2$, & $\overline{EH \times HG}$ ſpeciebus $a^2 - x^2$, $bx - x^2$ ſubſtitutis) $a^2 - x^2 : bx - x^2 = R : S$. Et dividendo (ſi R major ſit S) $a^2 - bx : bx - x^2 = R - S : S$. Ac proinde $\frac{S}{R-S} \overline{a^2 - bx} = bx - x^2$. Et $\frac{S}{R-S} a^2 = b + \frac{S}{R-S} b \times x - x^2$. Ad rectam igitur magnitudine datam $b + \frac{S}{R-S} b$ ſeu $EG + \frac{S}{R-S} EG$, ſpatium datum $\frac{S}{R-S} a^2$, ſive

[a] Ita, ni fallor, ſentiebant Veteres, qui quoties id agendum erat, ut data recta certâ aliquâ lege divideretur, ſegmenti utriuivis incogniti longitudinem computo quærebant. Dein ſedulo operam dabant. ut longitudinem computo definitam, Graphicè, abſque computo, invenirent. Huic labori reſpondet *æquationum conſtructio*, quam (quadraticas quod attinet) ſummo artificio veteribus tractatam eſſe, ſunt quæ fidem faciunt. Libri περὶ τομῆς διωρισμένης hujuſce argumenti toti erant.

$\frac{S}{R-S}$ EF², ut quadrato deficiat, applicando, definietur rectæ AB una atque altera longitudo.

Si æquales ſint R, S, erit $a^2-x^2=bx-x^2$, & $a^2=bx$. Et x, ſeu AB, $=\frac{a^2}{b}=\frac{EF^2}{EG}$.

Denique ſi S major ſit quam R, invertatur analogia primum expoſita ($a^2-x^2 : bx-x^2=R : S$,) ut fiet $bx-x^2 : a^2-x^2=S : R$. Dividendo $bx-a^2 : a^2-x^2=S-R : R$. Ac proinde ducendo extrema & media in invicem, & rite ordinando, $\frac{R}{S-R}bx + x^2 = a^2+\frac{R}{S-R}a^2$. Rectæ igitur datæ $\frac{R}{S-R}b$ datum rectangulum $a^2+\frac{R}{S-R}a^2$ applicando, ut quadrato excedat, definietur x ſeu AB. Datâ vero baſi AB, propter $\overline{AB+CD}$ ex hypotheſi datam, dabitur CD. Datis vero AB, CD, & angulo C, triangulum conſtruetur per vias Geometris notiſſimas.

LIMITES.

Propter triangulum, neceſſe eſt ut $\overline{AC+CB}$ majores ſint ſimul quam AB; ſive EF, ſeu a, major quam x. In æquationibus igitur expoſitis, in caſu ſecundo & tertio, ſcripto a ubique vice x, nullo fere negotio, prodibunt Limites.

In

In ſecundo, a^2 minus erit quam ba, ſeu b major quam a.

In 3°, ſubſtituto a pro x prodibit quoque limes, ba majus eſſe quam a^2. Quapropter & b majorem quam a.

In caſu autem primo, ut datum ſpatium $\frac{S}{R-S}a^2$ ad datam rectam $b+\frac{S}{R-S}b$, cum defectu quadrati, applicari poſſit, neceſſe eſt ut a major non ſit quam

$$\tfrac{1}{2}b\sqrt{\frac{R^2}{RS-S^2}}.$$

Porro ratio R ad S, ſive $\overline{AC+CB}^2-AB^2$ ad $\overline{AB\times CD}$, majoris eſt ad minus, ſi angulus datus ACB rectus ſit, vel recto minor, vel ſi obtuſus ſit, modo talis ſit magnitudinis, ut Radius ad tangentem ejus minorem rationem habeat quam numeri ternarii ad quaternarium. Quod ſi Radius ad tangentem Anguli ACB, obtuſi exiſtentis, rationem habeat eam quam numerus ternarius ad quaternarium, ratio R ad S æqualis erit ad æquale. Denique ſi Radius ad tangentem anguli, ACB, obtuſi rationem habeat eâ quæ eſt ternarii ad quaternarii majorem, ratio R ad S minoris erit ad majus. Radius vero ad tangentem anguli obtuſi ACB rationem habet quam 3 ad 4, ſi ſegmenti circularis, angulum ACB capientis, altitudo parti quintæ Diametri æqualis ſit.

Negotio perquam ſimili, alia quamplurima de triangulis problemata, quæ in *Arithmeticâ Univerſali* occurrunt, ope propoſitionem 66, 67 datorum, & earundem con-

converſarum, facile eſſet abſolvere; & generalius quidem quam a NEUTONO propoſita ſunt, pro angulo recto, dato quocunque ſubſtituto. Propoſitiones vero iſtas Datorum, difficillimis de Triangulis problematibus inſervientes, aut parum intellexiſſe, aut omnino neglexiſſe videntur Geometræ recentiores. Quo tamen crimine ſapientiſſimum illum NEUTONUM minime includo, Matheſeos, atque univerſæ adeo Philoſophiæ æternum SOLEM. Neque illi vitio dandum, quod in opere laudato, miſſâ quandoque Geometricâ indagine, problematibus nonnullis, quibus Geometricè facilius poſſet, Algebræ ſolius ope ſatisfecit. Algebræ utique dum præcepta traderet, æquum erat, ut non ſolum oſtenderet, qualis optimus eſſet, in rebus Geometricis, Algebræ uſus, (id quod pluribus exemplis indicare minime neglexit) ſed et quid ars Algebraica per ſe præſtare poſſet: et problematum ejus de Triangulis ipſius ſolutiones, id ſane præclari habent, quod exitus calculi conſtructiones indigitat, aliunde quam ex calculo, haud ita facile hauriendas.

Exempla nobis ultima ex ipſo Apollonii problemate de duobus ſemicirculis petita ſunto.

Semicirculi duo ADB, AEC quorum baſis CAB, in A contingant, neutro alterum includente. (Vid. Tab. 10. fig. 1.) Circumferentiis convexis interjectam puta ED, datæ magnitudinis, ad B vergentem, quæ producta circumferentiæ AEC iterum occurrat, ut in F. Jungatur AD, & â puncto F in baſin AC perpendicularis deducatur FG.

Rec-

Rectangulum $\overline{FB \times BE}$ rectangulo $\overline{CB \times BA}$ æquale. Et propter angulos ad D & G rectos, $\overline{FB \times BD} = \overline{BG \times BA}$. Ac proinde $\overline{FB \times ED} = \overline{CG \times BA}$. Quare ED : BA = CG : FB. Data igitur eſt ratio CG ad FB. Data igitur & ratio CG^2 ad $\overline{FB^2}$, ſive CG^2 ad $GB^2 + GF^2$, ſive denique CG^2 ad $BG^2 + \overline{CG \times GA}$. Data vero puncta C, A, B. Eo igitur res redit, ut datis, in rectâ poſitione datâ, punctis tribus C, A, B, quartum G in CA, inveniatur, ut CG^2 ad $BG^2 + \overline{CG \times GA}$ datam rationem habeat. Hoc vero Calculi negotium eſt. Pono igitur $CA = a$, $CB = b$. $CG = x$. Unde $GA = a - x$. $GB = b - x$. $AB = b - a$. Data denique ratio CG^2 ad $BG^2 + \overline{CG \times GA}$ eſt ED^2 ad AB^2. Quare $x^2 : b^2 - 2bx + x^2 + ax - x^2$, id eſt $x^2 : b^2 - 2bx + ax = ED^2 : AB^2$. Et $x^2 = \frac{ED^2}{AB^2} \times \overline{b^2 - 2bx + ax}$. Ac proinde $\frac{ED^2}{AB^2} \overline{2b - a} \times x + x^2 = \frac{ED^2}{AB^2} b^2$. Ad rectam igitur datam $\frac{ED^2}{AB^2} \overline{2b - a}$ applicetur ſpatium $\frac{ED^2}{AB^2} b^2$, ut excedat quadrato, & exceſſûs latitudo æqualis erit quæſitæ x ſeu CG. Data autem CG magnitudine, propter poſitione quoque datam, datum erit punctum G. Perpendicularis igitur GF poſitione data. Interſectio igitur ejus cum datâ circumferentia AEC, ſeu punctum F datum. Data igitur DF poſitione.

COMPOSITIO.

Detur recta V intra Limitem quem debet, fiat $BL = AB$. Ita erit $CL = 2CB - CA = 2b - a$.

Baſi

Baſi AB deſcribatur triangulum æquilaterum AMB, & MC, ML jungantur. Rectis AB, V, tertiam cape proportionalem, cui æquales ponantur MO, MP. Juncta PO rectis ML, MC in Q, R, producta occurrat. Erit CL, RQ = BM : MP = AB^2 : V^2 (ſive ED^2 in Analyſi) ac proinde erit RQ = $\frac{ED^2}{AB^2}$ $\overline{2b-a}$. Sed & CB : RP, ſive CB^2 : $\overline{CB \times RP}$ = MB : MP = AB^2 : ED^2. Quare $\overline{CB \times RP} = \frac{ED^2}{AB^2} CB^2 = \frac{ED^2}{AB^2} b^2$. Ad rectam igitur QR, applicetur $\overline{CB \times RP}$, ut quadrato excedat. Exceſſûs latitudo eſto SR. Fiat CG = SR. A G duc GF normalem. Junctâ BF problemati ſatisfeceris.

Semicirculis duobus AEB, CFD, in baſi BACD (vid. Tab. X. fig. 2,) poſitione datis, qui nullibi occurrant, neutro incluſo, circumferentiis convexis interjectam puta EF, datæ magnitudinis, ad angulum baſeos extremum B vergentem. Occurrat BF circumferentiæ CFD iterum in G. Ducatur GH ad baſin normalis, & jungatur AE. Propter circulum CFD, rectangula $\overline{GB \times BF}$, $\overline{DB \times BC}$ ſunt æqualia. Sed propter angulos ad E, H, rectos, rectangula $\overline{GB \times BE}$, $\overline{HB \times BA}$ æqualia. Sive $\overline{GB \times BE} = \overline{DB \times BA} - \overline{DH \times BA}$. Ac propterea $\overline{GB \times FE} = \overline{DB \times AC} + \overline{DH \times BA}$. Fiat ut AB ad AC ita DB ad DK. Datum erit punctum K, & rectangula $\overline{AC \times DB}$, $\overline{AB \times DK}$ erunt æqualia. Ac proinde $\overline{GB \times FE} = \overline{AB \times HK}$, & EF : AB = KH : GB. Data igitur eſt ratio KH ad GB, necnon KH^2 ad GB^2, ſive KH^2, ad $BH^2 + HG^2$, ſive denique KH^2 ad $HB^2 + \overline{DH \times HC}$. Eo igitur res redit, ut datis, in rectâ poſitione datâ, punctis quatuor K, D, C, B, quintum H inter D, C in veniatur

veniatur, quod faciat KH² ad HB² + $\overline{DH \times HC}$ in datâ ratione EF² ad AB². Factum puta, & eſto H punctum quæſitum. Poſitis igitur DC = a, DB = b, KD = c, et DH = x, per operam Algebraicam ſatis obviam invenio $\frac{EF^2}{AB^2}\overline{2b-a} + 2c.x + x^2. = \frac{EF^2}{AB^2}b^2 - c^2$. Ad datam igitur rectam $\frac{EF^2}{AB^2}\overline{2b-a} + 2c$ datum rectangulum $\frac{EF^2}{AB^2}b^2 - c^2$ applicando, ut quadrato excedat, definietur magnitudo rectæ DH. Datum igitur punctum H. Data igitur recta HG poſitione. Datum igitur punctum G, & recta BG poſitione data.

In haſce, & hiſce ſimiles caſuum reliquorum problematis Apolloniani reſolutiones, â prioribus, nec minus a Ghetaldi & Anderſoni [a] diverſas, ipſe olim incidi, antequam vel Pappi Lemmata evolverim, vel Ghetaldi aut Anderſoni Libros inſpexerim; quæ tamen Apol-

[a] Poſt editam, a Ghetaldo, Apollonii ſui Redivivi partem primam, in quâ problemata primi ſolummodi Libri reſoluta dedit, cum Viro præclaro, publicis negotiis tunc temporis implicito, legationem ſcilicet a municipio ſuo Ragufino apud Turcarum Imperatorem obeunti, problemati de duobus ſemicirculis vacare otium non ſuppeditaret, in lucem prodiit Alexandri Anderſoni Abredonenſis Libellus, *Supplementi Apollonii Redivivi* titulo ornatus, qui problematis illius reſolutiones continebat, reverâ, Algebraicas, ſed charactere proprio exutas, ut inanem aliquam præ ſe ferrent priſcæ illius Geometriæ ſpeciem. Quales autem fuerint, cum Anderſoni Libellum hodie paucis, credo, vidiſſe contigerit, exemplo, e facilioribus, allato, viſum eſt breviter indicare. In Tab. VIII. fig. 1. Propter triangula BML, BEA ſimilia, BM : BE = LB : BA. Ergo $\overline{BM \times BA} = \overline{LB \times BE}$. Ponatur BE = x. EF = c. DB = a. AB = b. AC = d. Erit LB = $a+d$. BC = $b+d$. Et $\overline{DB \times BC} = ab+ad$. Et BG = $\frac{DB \times BC}{BF} = \frac{ab+ad}{c+x}$ Ergo BM = $\frac{ab+ad}{c+x} + c$. Quare (propter $\overline{BM \times BA} = \overline{LB \times BE}$) $\frac{ab^2+abd}{c+x} + bc = ax + dx$. Multiplicando igitur per $c+x$, & rite ordinando, orietur æquatio quadratica $\overline{a+d}.x^2 + \overline{a+d-b}.cx - ab^2 - abd - bc^2 = o$. In æquationibus hujuſcemodi eruendis, & per ambages odioſas reſolvendis, Anderſonus totus eſt. Conſultius longe Ghetaldus, qui eo ſemper Calculum intendebat, ut rectarum BF vel BG longitudines erueret. Nec longe ei abfuiſſet, quin in conſtructiones noſtras, aut noſtris ſimillimas, incidiſſet, ni hominem Arithmeticæ gnarum, in re graphicâ plane idiotam, æquationum ſuarum Conſtructiones optimæ ſemper latuiſſent.

lonianis

lonianis quam non ſunt conferendæ ſentio, nec invitus fateor.

Neſcio an operæ pretium ſit, in eodem problemate, viam indaginis aliam inſuper monſtrare, latam ſane & patentem, ſed a nemine ante nos tritam, poſt duas Ghetaldi & Anderſoni proprias, duaſque alias a nobis jam ſupra oſtenſas, ſolam, quantum ſentio, reliquam, quæ ad compoſitiones, haud inconcinnas, per prima duo libri 1. problemata recto tramite perducit. In Tab. VIII. fig. 1. definiendam ſumo rectæ FG magnitudinem.

Pono igitur $KE=x$. $FE=c$. $HA=a$. $AB=b$. $AC=d$. Erit igitur $BC=b+d$. $DA=2a-d$. Et $DB=2a+b-d$. Et propter HK, AE, parallelas, $KE : EB=HA : AB=a : b$. Quare $EB=\frac{KE\times b}{a}=\frac{bx}{a}$. Et $BF=\frac{bx+ac.}{a}$ Sed propter circulum DFC, $BG=\frac{DB\times BC}{BF}=\frac{2a^2b+2a^2d+ab^2-ad^2.}{bx+ac}$ Quare $BM=\frac{2a^2b+2a^2d+ab^2-ad^2}{bx+ac}+c$. Sed propter ML, EA parallelas $BM : BE=LB : BA$. id eſt, $\frac{2a^2b+2a^2d+ab^2-ad^2}{bx+ac}+c:\frac{bx}{a}=2a+b : b$. Ducendo igitur extrema & media in invicem, fractiones per multiplicationes, quas oportet, tollendo, & rite ordinando, hæcce orietur æquatio.

$2a^3b+2a^3d+a^2b^2-a^2d^2+a^2c^2=\overline{2ab+b^2}.x^2+2a^2cx.$

Et dividendo per $2ab+b^2$

$\frac{a^2}{2ab+b^2}\overline{2ab+2ad+b^2-d^2+c^2}=\frac{2a^2}{2ab+b^2}cx+x^2.$ Id eſt

eſt $HA^2 + \frac{HA^2}{LB \times BA} \overline{LC \times CA} + \frac{HA^2}{LB \times BA} EF^2 =$ $\frac{HA^2}{LB \times BA} 2EF.x + x^2$. Cape igitur $\sqrt{LB \times BA}$ quæ dicatur M, et $\sqrt{LC \times CA}$, quæ dicatur N. Fiat ut M ad HA ita N ad quartam, quæ dicatur P, et ut M ad HA ita EF ad quartam, quæ dicatur R. Et fiat ut EF ad R ita R ad tertiam, quæ dicatur S. Ad rectam 2S, ſpatium $HA^2 + P^2 + R^2$ applicando, ut quadrato excedat, definietur x ſeu KE. Datâ autem KE, propter datam EF, dabitur KF, ac proinde 2KF, ſeu FG. Datâ vero FG, problema componetur ope prop. 1. Lib. 1.

Coeant puncta C, A, ut circuli extrinſecus contingant, ut in Tab. III. fig. 7. Coeunt igitur L, D, puncta, & d æquatur nihilo. Unde æquatio ſupra expoſita in hancce migrat. $\frac{a^2}{2ab+b^2} \overline{2ab+b^2+c^2} = \frac{2a^2}{2ab+b^2} cx + x^2$. Id eſt $HA^2 + \frac{HA^2}{DB \times BA} EF^2 = \frac{2HA^2}{DB \times BA} EF.x + x^2$ Sive in Tab. III. fig. 7, $KA^2 + \frac{KA^2}{CB \times BA} ED^2 = \frac{KA^2}{CB \times BA} 2ED.x + x^2$. Cape igitur $M = \sqrt{CB + BA}$. Fiat M : KA = ED : R. Et ED : R = R : S. Ad rectam 2S ſpatium $KA^2 + R^2$ applicando, ut quadrato excedat, dabitur x ſeu LD. Inde dabitur LE, & 2LE ſeu FE, & per prop. 1^m. Libri 1. problema componetur.

FINIS.

MVSEVM
BRITAN
NICVM

Lib. II.
Tab. IX.

Fig. I.

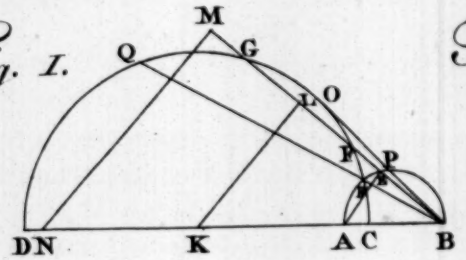


Fig. 2.

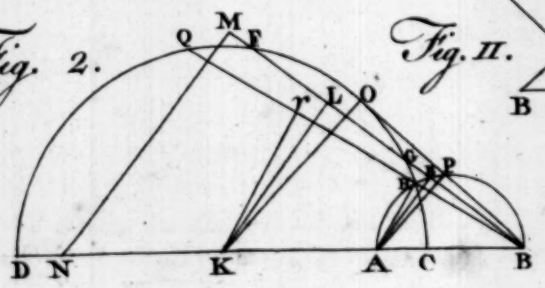


Fig. II.

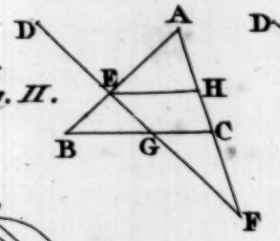


Fig. 12.

Fig. 3.

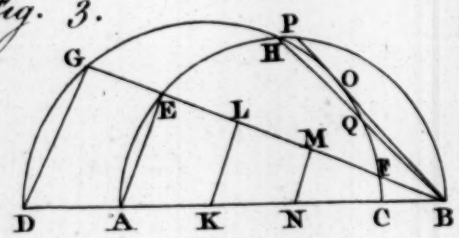


Fig. 4.

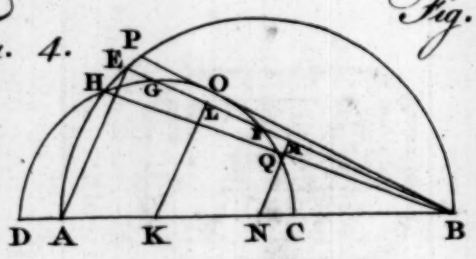


Fig. 9.

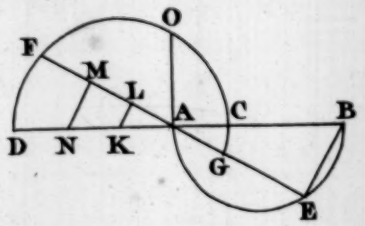


Fig. 5.

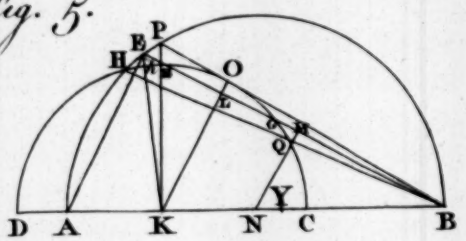


Fig. 6.

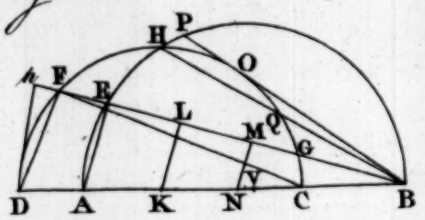


Fig. 10.

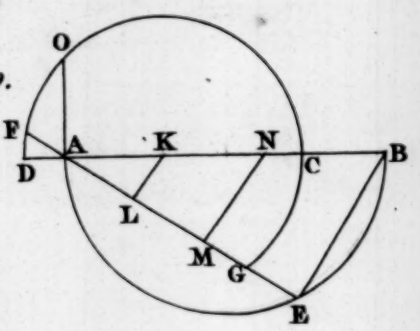


Fig. 7.

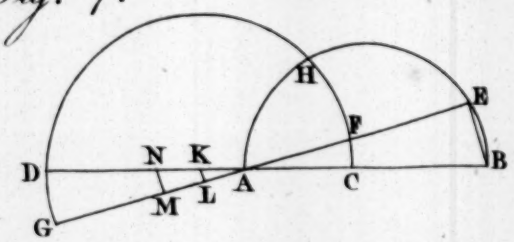
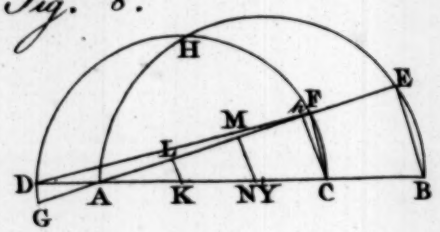


Fig. 8.



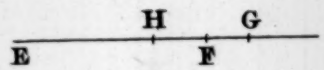
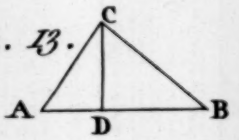


Fig. 13.



Lib. II.
Tab. X.

Fig. 1.

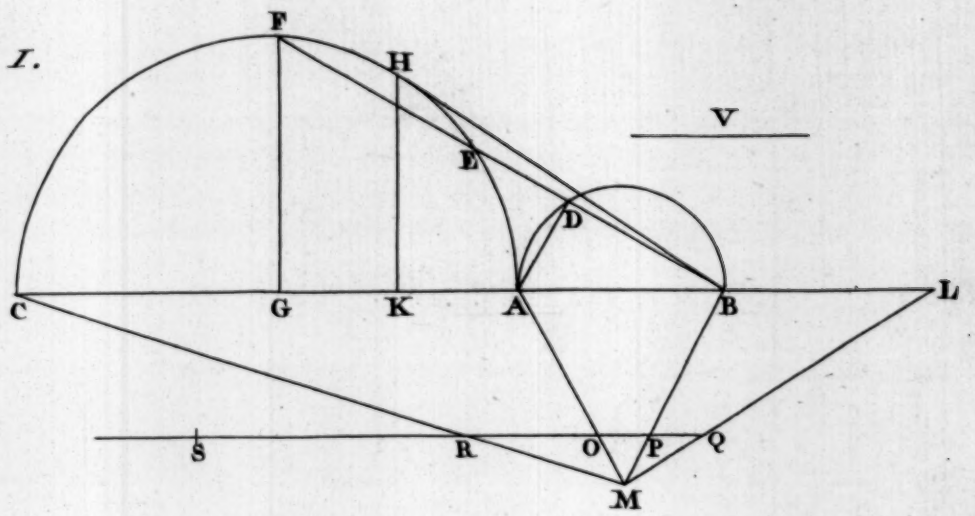
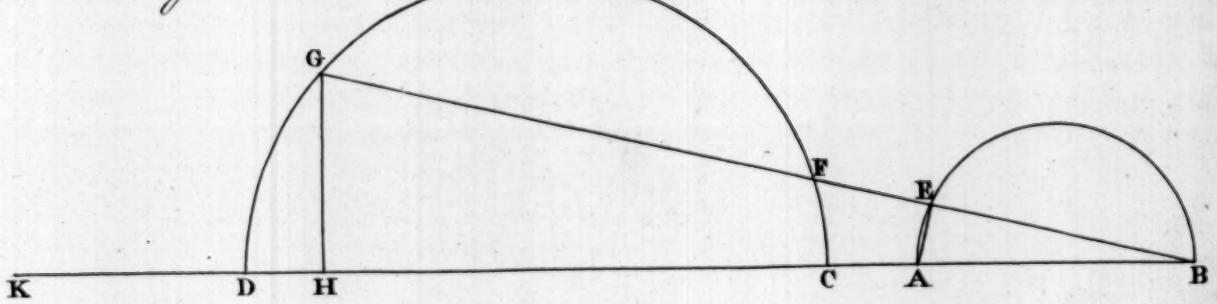



Fig. 2.